Relative Performance, Banker Compensation, and Systemic Risk

Rui Albuquerque  
*Boston College*  

Luís Cabral  
*New York University*  

José Corrêa Guedes  
*Católica Lisbon School of Business and Economics*

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**Abstract.** This paper shows that in the presence of correlated investment opportunities across banks, risk sharing between bank shareholders and bank managers leads to compensation contracts that include relative performance evaluation and to investment decisions that are biased toward such correlated opportunities, thus creating systemic risk. We analyze various policy recommendations regarding bank managerial pay and show that shareholders optimally undo the intended risk-reducing effects of the policies, demonstrating their ineffectiveness in curbing systemic risk.
1. Introduction

Central banks around the world are entering unchartered territory by regulating pay of bank Chief Executive Officers (CEOs). These actions are a response to the view that bank executives’ compensation packages are one of the main culprits of the risk taking in the banking industry that preceded the recent financial crisis (e.g., International Monetary Fund, 2014).\footnote{There is a debate on the link between compensation and risk taking. Bebchuk et al. (2010), Bhagat and Bolton (2013), DeYoung et al. (2013) and Cai et al. (2010) have argued that the incentive component of pay may have caused excessive risk taking. In contrast, Cheng et al. (2015), Fahlenbrach and Stulz (2011), and Hagendorff et al. (2016) have disputed the link between firm risk and CEO compensation.} Loosely speaking, excessive risk can arise if bank CEOs are shielded from significant negative shocks to their own banks because of poorly designed compensation packages (e.g., Admati and Hellwig, 2013; International Monetary Fund, 2014; and Geithner, 2010). This paper provides a model with a novel mechanism through which pay of bank executives can lead to systemic risk in the banking industry as it leads banks to take on correlated actions. It then uses the model to analyze the effectiveness of many of the new regulatory actions by central banks in reducing systemic risk.

The question we ask is whether optimally designed compensation packages, that are not misaligned in any way by managerial entrenchment, can lead to systemic risk even in the absence of bailout guarantees. The goal is to identify contractual features in compensation that can potentially lead to systemic risk and that thus may warrant pay regulation by a central bank who values social welfare losses from systemic risk.

We model two identical banks each bank with a risk-neutral principal (the shareholders) and a risk-averse agent (the CEO). Each bank has access to two investment opportunities, one with only idiosyncratic risk and another that carries risk that is correlated across banks. The agent is required to spend costly unobservable effort to increase the return of the projects available to the bank and makes an unobservable portfolio allocation of how much of each investment opportunity to pursue. To focus on risk alone we assume equal expected returns to both projects and thus an equal contribution of effort to expected returns.

As in the classical principal-agent setting with hidden action, in our model the agent is induced to deploy unobservable effort by linking her pay to the bank’s performance. However, because the agent is risk-averse, this contract can be improved by reducing the volatility in the compensation of the manager by incorporating relative performance evaluation (RPE). Having compensation depend on relative performance rather than on absolute performance works to reduce volatility of pay and is particularly effective when there is a high degree of correlation among the performance of the bank with its rival.

The novelty in the model arises from the strategic interactions between the two banks and the endogeneity of the industry return. The presence of relative performance in the compensation scheme leads the manager to choose to put more weight on investments that are common to the rival bank, as opposed to bank-specific invest-
ments subject to idiosyncratic risks. In addition, the weights placed by each bank in the common project are strategic complements. The more one bank chooses to invest in the common project the greater the correlation of the banks’ overall returns if the other bank also chooses to invest more in the common project. With greater correlation comes less overall risk in pay for the same amount of relative performance in each contract. In turn, this gives rise to a strategic complementarity in the amounts of RPE in the compensation of the managers of the two rival banks: if one bank designs a compensation package with more RPE, the optimal response of the shareholders of the rival bank is to increase the RPE in the compensation of their own manager. With more relative performance and a greater weight on the common project, the manager’s pay volatility decreases but at the cost of an increasing amount of systemic risk associated with the increased likelihood of joint bank failure that comes with the greater investment in the common project.

We then extend the model to allow for bank leverage. We show that with leverage, the manager is incentivized to invest more in both risky projects, as these earn a return higher than the borrowing rate. Because some of the risk associated with the correlated project can be hedged via RPE, the manager is offered more RPE, and engages in relatively more investment in the correlated project, than in the model without leverage.

Although we offer a very specific rationale for RPE, several of our results are also consistent with other motivations for RPE. For example, shareholders may resort to RPE as a means to attract higher-ability managers. However, given that RPE is in place (and regardless of the reasons for its existence), risk averse bank managers will tend to choose common assets, thus creating systemic risk.

The model offers several predictions. First, RPE in executive compensation should be common in banking, allowing shareholders to grant more powered incentives that lead CEOs to work harder, increasing bank productivity and returns. While the earlier literature of RPE produced mixed results across industries, more recent evidence from both the implicit and explicit use of RPE suggests that the generality of firms use RPE in CEO pay (see, for example, Albuquerque, 2009, on implicit RPE and Angelis and Grinstein (2016), on explicit RPE). Finance, in particular, has been found to be an industry where RPE is pervasive: Albuquerque (2014) estimates that the finance industry has one of the highest average levels of RPE in CEO pay, second only to utilities firms; Angelis and Grinstein (2016) find that 37% of firms in their Money industry subsample use RPE; they also find that the Money industry is in the top 25% in terms of intensity of RPE use; Ilic et al. (2015) examines the usage of RPE in a sample of non-US large international banks, finding that 60% disclose the usage of RPE and that the likelihood of RPE adoption increases with bank size. The usage of RPE in banking has also been shown to have increased following the deregulation of banking in the early eighties, accompanying a parallel increase in the pay-for-performance sensitivity of bank CEOs (Crawford, 1999). Moreover, as predicted by our model, empirical studies on implicit RPE usage uncover evidence of RPE only when peers are chosen narrowly to capture firms exposed to similar exogenous shocks
Second, the usage of RPE in the pay of bank executives should be accompanied by herding in the choice of risk exposures across banks, creating systemic risk. In line with this prediction, Bhattacharyya and Purnanandam (2011) report that between 2000 and 2006 — that is, the period preceding the financial crisis — the idiosyncratic risk of US commercial banks dropped by half, whereas the systematic risk doubled. This prediction is shared with the models of Acharya and Yorulmazer (2008) and Farhi and Tirole (2012) because there, too, an implicit bailout guaranty leads banks to take on correlated risk. Third, executive pay volatility decreases as industry volatility increases on account of the RPE effect. This prediction is new as it related directly to executive pay as a source of systemic risk: it can help identify our mechanism from other sources of systemic risk like bailout guarantees. Fourth, the endogenous variables of our model — intensity of incentive pay, intensity of RPE, degree of herding in bank risk exposures and amount of systemic risk — should vary over time as a function of the availability of correlated projects. In particular, the lowering of barriers to bank competition (such as regulatory impediments to competition across different geographies business lines or, yet, impediments to international trade) that enhance the creation of a unified global banking market, should produce more extreme outcomes for the model’s endogenous variables. Fifth, leverage magnifies the benefit of RPE, resulting in more RPE and greater systemic risk.

The second part of the paper takes a normative perspective, examining how different constraints on the compensation of bank executives either already adopted or currently being considered by regulators affect the equilibrium of the model — i.e., the endogenous optimal compensation package of managers and the endogenous optimal structure of banks’ investment portfolio — and the level of systemic risk resulting thereof. In this regard, we argue that without a regulatory constraint on the amount of RPE received by bank executives, some of the restrictive measures on executive compensation that are usually considered by regulators are ineffective in reducing systemic risk. For example, ignoring the effect of leverage, imposing a cap on equity incentives leads banks in the model to change the amount of relative performance pay in such a way as to keep incentives unchanged regarding the amount invested in

2. A related issue is whether RPE determines management turnover. Barro and Barro (1990) and Barakova and Palvia (2010) find that RPE plays an important role in the dismissal decisions of bank executives. Barakova and Palvia (2010), however, document that, in an industry downturn, absolute performance plays a more important role than relative performance in determining executive turnover, a result which they interpret as evidence that “bad times reveal the quality of management.”

3. Brunnermeier et al. (2012) documents that banks increased their income from trading, investment banking and venture capital income, all noncore, nontraditional income, and those relying more heavily on these sources of income contributed to a greater extent to systemic risk.
the common project and hence the amount of systemic risk. On top of the inability to affect systemic risk, an unintended consequence of a cap on equity incentive pay is the reduction of the amount of managerial effort and thus on productivity in the industry.

When we consider the additional effect of leverage, we find that a cap on incentive pay has the perverse effect of increasing leverage (as the manager becomes less responsive to the risk of the bank’s return), raising the amounts invested in all risky projects, especially the correlated project due to the beneficial effect of RPE. In sum, a cap on pay increases systemic risk because of the leverage effect. Not surprisingly then, we show that a cap on leverage can be effective at curbing systemic risk.

We view this ineffectiveness result as a formalization of the argument put forth in Posner (2009, p. 297) that “Efforts to place legal limits on compensation are bound to fail, or to be defeated by loopholes, or to cause distortions in the executive labour market and in corporate behaviour.” More than a “loophole,” we argue that existing dimensions of executive pay will adjust to an artificial regulation of one dimension in isolation; and that, as a result, no positive effect will take place in terms of systemic risk; rather, a negative effect (a “distortion”) may take place in “corporate behavior.” Murphy (2009) and Ferrarini (2015) hypothesize unintended consequences of regulating executive pay on the quality of the workforce and the productivity of the industry. Kleymenova and Tuna (2015) provide evidence that an unintended consequence of the increased regulation in the U.K. is that compensation contracts have become more complex for U.K. banks relative to other firms in the U.K. In the same spirit, French et al. (2010) suggests that governments should not regulate the level of executive pay in financial firms because markets are better at setting prices.

**Literature Review.** A large literature examines the motivations for herding in managerial decisions. Within this literature only a few authors study the choice of projects or business activities by banks and the systemic risk resulting from correlated choices, but none that we know go on to study the implications of constraining parameters of the compensation contract.

The papers that are closer to us associate endogenous executive compensation with endogenous investment choices. Zwiebel (1995) assumes that managers have private information about their ability and make an unobservable choice between a standard industry project and a non-standard project that delivers a higher mean return. Relative performance evaluation in managerial pay filters out from realized project returns systematic industry factors, thus improving the inference with respect to managerial quality particularly so if the manager chooses the standard project. Zwiebel shows that managers of average quality herd in the standard project while managers of either high or low ability choose the non-standard project. Ozdenoren and Yuan (2014) analyze an industry populated by a continuum of principal-agent pairs, where each pair faces a classical moral hazard problem. They assume that the return obtained by each pair depends on the effort made by the agent and on an unobservable aggregate shock in a multiplicative fashion, and on a firm-specific shock. The aggregate return
therefore equals the aggregate shock times the average effort level in the industry. As in Zwiebel, the closer the agent’s effort level is to the industry’s average, the more informative is the industry return and the more valued is relative performance evaluation. The main difference with our setting is that in Ozdenoren and Yuan the choice of risk is tied to the choice of return; agents’ effort choices become correlated and systemic risk is higher when expected industry productivity is high. In contrast to Zwiebel and Ozdenoren and Yuan, in our setting correlated strategies are optimal even when expected returns are equated across projects.

Maug and Naik (2011) and Gumbel (2005) show that fund managers compensated with relative performance contracts engage in correlated strategies. Maug and Naik (2011), like Zwiebel, do not endogenize the contract terms when discussing firm strategies, and would therefore be limited in analyzing how the equilibrium changes in response to pay regulation. In Gumbel, it is the principal that chooses both the contract terms and which assets to invest in. This is a somewhat less reasonable assumption in the context of sequencing of decision making in banks. Bhattacharya et al. (2007) model relative performance in contracts only on the bank’s upside and find that it leads banks to specialize, rather than take on correlated projects. In Buffa et al. (2014) RPE arises as a way to alleviate agency frictions. We have purposefully left agency considerations out so as to steer the discussion to risk sharing. Relative to the above papers, we allow our firms to hold leverage, an important feature of banks that we show to be influential in the policy discussion.

Another set of papers focuses on government guarantees and their role in creating incentives for banks to choose correlated strategies (Kane, 2010). In Acharya and Yorulmazer (2008) the benefit of engaging in correlated strategies arises when banks are underperforming and the central bank bails them out. The cost of engaging in correlated strategies is the additional rent that can be garnered by a surviving bank after buying the failed bank. In Farhi and Tirole (2012), the time consistent decision of the banking regulator is to bailout banks in the event of a shock if the extent of the banking crisis is big enough. This regulatory moral hazard makes banks’ choices of balance sheet risk strategic complements and banks take on correlated risks. Acharya et al. (2015) model a risk shifting problem when there is too much debt and an inadequate loan monitoring problem when there is too little debt. They show that bailout guarantees can arise in an equilibrium where banks take on excessive debt, engage in risk shifting, and fail together. Our model does not require bailout guarantees to generate systemic risk, but bailout guarantees would magnify the mechanism we describe by increasing the benefit from using RPE. Our paper points to optimal private incentives to generate systemic risk in the absence of a regulator.

Other, less related mechanisms have been suggested as a way to generate correlated choices of agents in the banking industry. Acharya and Yorulmazer (2008) model banks that in order to minimize their cost of borrowing seek to minimize the information content about their exposure to systematic risk conveyed by the performance of rivals’ loan portfolio. They show that the optimal bank strategy is to undertake correlated investments. In Acharya (2009), the failure of one bank entails
a recessionary spillover on surviving banks, creating an incentive among banks to fail and survive together. Allen et al. (2012) propose a model where banks diversify their idiosyncratic risks by swapping assets. There is an equilibrium clustered structure where banks hold correlated assets. Imperfectly informed creditors do not roll over short term debt in the presence of adverse signals and banks default together. Martinez-Miera and Suarez (2014) study dynamic incentives of banks and show that correlated strategies, which yield higher returns in good states, are more likely to occur after extended good aggregate periods that allow banks to accumulate capital to be used to meet potential future capital regulatory constraints. In Wagner (2010), diversification is costly because it increases the odds that any two banks are invested in the same sector, making fire sales more costly in distress in that sector.

There is a growing literature that studies the effects of constraints on executive pay in various settings. Most of these papers are cast in the context of a single-bank model and thus fail to take into account strategic effects across banks in their design of compensation. Moreover, to the best of our knowledge, none of the existing papers disentangles idiosyncratic risk from systemic risk, a focal point of our paper. For “too-big-to-fail” institutions bank-specific risk may be equated to systemic risk. Our focus on correlated actions as the driver of systemic risk points to a complementary concern for regulators, one that we show is intertwined with contractual features in executive compensation. Specifically, we argue that to evaluate whether risk taking at the level of individual banks translates into systemic risk one has to determine whether the risks taken by banks, large and small, are diversifiable at the industry level. Hence our choice is to model an industry equilibrium. Several papers describe scenarios where it is optimal to impose constraints on one or more of the components of executive pay (see, for example, John and John, 1993; Bolton et al., 2015; Edmans and Liu, 2011; Thanassoulis, 2012, 2014; Chaigneau, 2013). Others highlight the risks and unintended consequences of several of the same constraints (e.g., Llense, 2010; Dittmann et al., 2011; Asai, 2016). Yet others point to the value of combining restrictions on pay with restrictions on other bank policies like leverage (e.g., John et al., 2000; Kolm et al., 2014; Hilscher et al., 2016).

Finally, our paper is related to a literature that studies spillovers in governance through compensation packages and the labor market for executives. As in our paper, Acharya and Volpin (2010) and Dicks (2012) show that compensation choices of firms are strategic complements and thus the weakening governance in one firm that raises pay to its CEO induces other firms to also raise pay to their CEOs and to weaken governance. Cheng (2011) shows that RPE can cause correlated choices in governance across firms when managers have career concerns. Levit and Malenko (2016) show that directors’ willingness to serve on multiple boards creates correlated choices in governance.

The structure of the rest of the paper is as follows. In Section 2 we present the ingredients of our basic model, which is solved in Sections 3 and 4. Section 3 examines the managers’ optimal choice of investment policies and effort while Section 4 analyses the shareholders’ optimal design of the compensation contracts. Section
extends the basic model to the case in which the banks are levered, with leverage being an additional endogenous variable determined by management. The models, with leverage and without, are used to cast light on the implications for systemic risk of regulatory constraints on the pay of top bank managers in Section 6. Section 7 concludes.

2. Model

Consider an industry with two banks, denoted \( i = 1, 2 \). Suppose that bank \( i \)'s CEO has a utility function \(- \exp(-w_i + d_i)\), where \( w_i \) is CEO compensation and \( d_i \) the CEO's disutility from effort \( e_i \). By assuming an exponential utility function, we assume back CEOs are risk averse. By contrast, we assume bank shareholders are risk neutral.

Compensation is a linear function of own and rival bank performance:

\[
    w_i = k_i + a_i r_i - b_i r_j
\]

where \( j \neq i \) and we assume \( a_i, b_i > 0 \) are compensation coefficients to be determined by shareholders as part of the CEO contract. In particular, \( b_i \) corresponds to relative performance evaluation, the central issue of our analysis.

We assume the CEO's disutility of effort is quadratic:

\[
    d_i = \frac{1}{2} \gamma_i e_i^2
\]

The bank's return, \( r_i \), is a combination of: effort, \( e_i \); return on an activity of a type that is available to the whole industry, \( c_i \); and return on an activity that is available to the bank alone, \( s_i \). Until Section 5 we exclude the possibility of leveraging. This implies that each bank's assets are equal to its equity; and the CEO's portfolio choice is limited to determining the fraction \( x_i \) of assets invested in common assets, where \( x_i \in [0, 1] \). We thus have

\[
    r_i = e_i + x_i c_i + (1 - x_i) s_i
\]

Since our focus is on risk and correlation induced by joint portfolio choices, we assume that all underlying assets have the same expected value and variance. Specifically,

4. We also assume that the coefficient of risk aversion is equal to 1. Our results can be generalized to bank CEOs with a coefficient of risk aversion equal to \( \eta \in \mathbb{R}_+ \).
5. We consider the shareholders' problem below. Our risk-neutrality assumption is not innocuous: the collapse of the banking system would have to be a risk that cannot be diversified away, whereas under the risk-neutrality assumption we implicitly assume that shareholders would be able to do so.
6. The optimality of linear contracts with relative performance is discussed for example in Dybvig et al. (2010). Below we discuss the implications of adding stock options to the contract.
7. In this setting, total risk is fixed; the CEO determines the composition of such risk.
we assume that $c_i$ and $s_i$ are normally distributed with mean $\mu$ and variance $\sigma^2$; and with no further loss of generally we assume $\sigma^2 = 1$.

Our crucial assumption regarding the underlying assets is that, while $s_1$ and $s_2$ are independent, $c_1$ and $c_2$ are positively correlated. Specifically, we denote by $\psi$ the covariance of $c_1$ and $c_2$ and assume that $\psi \in [0, 1]$. We also assume that $s_i$ is independent of $c_i$ (as well as $c_j$ and $s_j$).

The timing of the game proceeds as follows. In a first stage, risk-neutral shareholders simultaneously determine their CEO’s compensation parameters: $k_i, a_i$ and $b_i$. We assume that $(k_i, a_i, b_i)$ is observed by bank $i$’s CEO but not by other banks. This assumption reflects the fact that compensation contracts are typically observed with considerable noise. Next, CEOs simultaneously choose effort $e_i$ and portfolio structure $x_i$. Finally, Nature generates the values of $c$ and $s_i$; and payoff as paid.

We derive the Nash equilibrium of this multi-stage game, providing conditions such that the equilibrium exists and is unique; and compare it to the benchmark where RPE is not present (that is, $b_i = 0$).

3. Portfolio choice without leverage

Substituting (2) for $r_i, r_j$ in (1), we get

$$w_i = k_i + a_i (e_i + x_i c_i + (1 - x_i) s_i) - b_i (e_j + x_j c_j + (1 - x_j) s_j)$$

(3)

It follows that the first and second moments of CEO compensation are given by:

$$\mathbb{E}(w_i) = k_i + a_i e_i - b_i e_j + (a_i - b_i) \mu$$

(4)

$$\mathbb{V}(w_i) = a_i^2 x_i^2 + b_i^2 x_j^2 - 2 a_i b_i x_i x_j \psi + a_i^2 (1 - x_i)^2 + b_i^2 (1 - x_j)^2$$

(5)

Since $w_i$ is linear in $r_i$ and $r_j$; and since the latter are normally distributed; it follows that the CEO’s utility maximization problem is equivalent to

$$\max_{e_i, x_i} \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2$$

(6)

The first-order condition with respect to $e_i$ is given by

$$a_i - \gamma e_i$$

and so

$$e_i^* = a_i / \gamma_i$$

(7)

where a * denotes optimal (or best-response) value. This is a standard principal-agent result: effort is increasing in performance evaluation and decreasing in the disutility of effort parameter. We next move to the CEO’s optimal portfolio choice. The first-order condition with respect to $x_i$ is given by

$$-a_i \left( a_i x_i - \psi b_i x_j \right) + a_i^2 (1 - x_i) = 0$$

(8)
(Notice the second-order condition is satisfied if and only if $a_i > 0$.) It follows that

$$x_i^* = \frac{1}{2} + \frac{\psi b_j x_j}{2 a_i}$$

(9)

If there is no RPE — that is, if $b_i = 0$ — then $x_i^* = \frac{1}{2}$. This corresponds to the standard result of risk lowering by portfolio diversification. Since the assets $c_i$ and $s_i$ are identically and independently distributed, it is optimal to split the portfolio equally across the two. By contrast, setting $b_i > 0$ induces a demand for hedging: by increasing the value $x_i$, bank $i$’s CEO decreases the variance of its compensation. An immediate implication of (9) is that

**Proposition 1.** $x_i^*$ is increasing in $x_j$.

The intuition is that, under relative performance evaluation (that is, with $b_i > 0$) choosing the common asset $c_i$ is a form of “insurance” by bank $i$’s CEO. Specifically, under relative performance evaluation, a high value of $c$ is bad news for firm $i$’s CEO to the extent that firm $j$’s CEO has chosen that asset. In order to hedge against this adverse outcome, bank $i$’s CEO optimally chooses to place a greater weight on asset $c$ as well. In other words, Proposition 1 states that $x_i$ and $x_j$ are strategic complements: bank $i$’s CEO benefits from investing in $c$ because bank $j$’s CEO does so. In fact, this allows us to characterize the equilibrium of the portfolio-choice game as well as its comparative statics with respect to performance evaluation parameters:

**Proposition 2.** If $a_i \geq b_i > 0$, then the portfolio-choice game has a unique equilibrium. Moreover, the equilibrium levels $\hat{x}_k$ are strictly increasing in $b_i$.

In other words, CEOs choose the common asset to the extent that rival CEOs choose the common asset and compensation is based on relative performance.

We now turn to the analysis of overall industry returns, which are given by

$$R \equiv \sum_{i=1,2} r_i = \sum_{i=1,2} (e_i + x_i c_i + (1 - x_i) s_i)$$

(10)

We define systemic risk as the variance of overall industry returns, $\mathbb{V}(R)$. The next result, which is a corollary of Proposition 2, characterizes $\mathbb{V}(R)$.

**Proposition 3.** An increase in $b_i$ leads to an increase in systemic risk.

In words, Proposition 3 encapsulates one of our main results: relative performance evaluation may lead to an increase in systemic risk. The irony of Proposition 3 is that the increase in overall risk results from the CEOs desire to reduce their individual risk. In fact an increase in relative performance pay decreases managerial pay risk while increasing systemic risk in the banking industry.
4. Corporate governance

We now take one step back and consider the optimal (and equilibrium) choices by shareholders. Bank $i$’s shareholders, who we assume are risk neutral, choose $k, a_i, b_i$ so as to maximize the expected value of $r_i - w_i$. Specifically, the maximization problem is given by

$$\max_{k, a_i, b_i} E(r_i - w_i)$$
$$\text{s.t. } E(w_i) - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i$$
$$e_i = e^*_i(a_i)$$
$$x_i = x_i^*(a_i, b_i; x_j)$$

(11)

Our first result in this section provides conditions such that relative performance emerges in equilibrium. First, we note that, from (9), portfolio choices are only a function of the ratio $p_i \equiv b_i/a_i$

That is, $p_i$ measures the intensity of relative performance evaluation at bank $i$. Given this definition, the best-response mapping (9) may be re-written as

$$x_i^* = \frac{1}{2} \left( 1 + \psi p_i x_j \right)$$

(12)

Notice that (12) confirms Proposition 3: an increase in relative performance by firm $i$ (measured by $p_i$) leads to an increase in $x_i$ and $x_j$: Equation (12) shows that the partial effect is to increase $x_i$; and supermodularity implies that both $x_i$ and $x_j$ increase in the resulting subgame equilibrium. As one would expect, if $p_i = 0$, then the CEO’s optimal portfolio choice is $x = \frac{1}{2}$: a mean-variance-utility CEO’s optimal portfolio is to place equal weights on i.i.d. projects.

Proposition 4. In equilibrium $a_i, b_i > 0$ (and so $p_i > 0$)

Risk-neutral shareholders are indifferent with respect to their bank’s portfolio composition. However, the need to compensate risk-averse CEOs leads shareholders to “internalize” the CEO’s risk aversion. Specifically, an increase in $b_i$ leads to a decrease in the variance of CEO pay, which in turn allows shareholders to lower base pay. In other words, the thrust of Proposition 4 is that shareholders are willing to go along with the CEO’s desire to reduce risk; and relative performance evaluation enables CEOs to follow a risk-reducing portfolio strategy.

This result is not an artifact of the linearity in the contract. Suppose shareholders were to give stock options to the CEO. Such options give incentives to build volatility in the firm’s own stock returns, which is accomplished by concentrating investments in any one of the two projects (since they are independent and have the same volatility). With RPE, CEOs would have a preference for the common project as it increases the correlation of returns and reduces the volatility of pay (while having no effect on the volatility of the underlying of the stock options). As shareholders want to reduce
volatility in pay, RPE is optimal. The optimality of RPE also holds if banks are held by a common risk-neutral shareholder. Common ownership does not alter the objective of minimizing the variance of pay of each bank’s CEO.

**Comparative statics.** Proposition 4 states that, in equilibrium, relative performance evaluation is enacted. However, it does not say much regarding the level of relative performance evaluation, \( p_i \equiv a_i/b_i \), or regarding the equilibrium portfolios chosen by bank managers. The following result addresses these issues:

**Proposition 5.** There exists a unique symmetric equilibrium. It has the property that \( x \) and \( p \) are strictly increasing in \( \psi \), ranging from \((p = 0, x = \frac{1}{2})\) when \( \psi = 0 \) and \((p = 1, x = 1)\) when \( \psi = 1 \). Moreover, if \( \psi < 1 \) then \( p < \psi \).

As expected, if \( \psi = 0 \), that is, if there is no correlation between the CEO’s outcome (even when they invest in the same asset), then there is no point in offering RPE \((p = 0)\): in fact, RPE would only add noise to the system without creating any additional incentive.

**The strategic nature of relative performance evaluation.** Earlier we showed that the portfolio \( x_i \) choices are strategic complements. A similar question may be asked regarding the choices of RPE, \( p_i \).

**Proposition 6.** There exist \( 0 < \psi' < \psi'' < 1 \) such that, if \( \psi < \psi' \) (resp. \( \psi > \psi'' \)), then \( p_1 \) and \( p_2 \) are strategic complements (resp. substitutes).

The simpler intuition for Proposition 6 corresponds to the case when \( \psi \) is small. When that is the case, an increase in \( p_2 \) leads to an increase in \( p_1 \): RPE choices are strategic complements. By (12), an increase in \( p_2 \) leads to an increase in \( x_2 \). Given that \( x_2 \) is greater, the potential for variance decrease by increasing \( x_1 \) is greater. As a result, the incentive for Bank 1’s shareholders to increase RPE also increase.

Formally, the proof of Proposition 6 develops along the following lines. As shown in the Proof of Proposition 4, the first-order condition for shareholder \( i \) payoff maximization with respect to \( b_i \) implies

\[
p_i = \frac{\psi x_i x_j}{x_j^2 + (1 - x_j)^2}
\]  

\( (13) \)

In other words, it’s as if shareholder \( i \) “anticipates” the values of \( x_i, x_j \) and, accordingly, adjusts the choice of \( p_i \). Now suppose that \( \psi \) is small, specifically close to zero. Then \( x_j \) is close to \( \frac{1}{2} \). It follows that a small change in \( x_j \) has little effect on the denominator of (13). Therefore, all of the action is in the numerator, which is increasing in \( x_i \) and \( x_j \). An increase in \( p_j \) leads to an increase in \( x_j \) (cf (12)), and supermodularity implies that \( x_i \) increases as well. Together, this implies an increase in \( p_i \). At the opposite extreme, if \( \psi \) is close to 1, then the denominator is increasing in \( x_j \) (at a high rate), which more than compensates for the increase in the numerator.
and implies that the increase in $x_j$ leads to a decrease in $p_i$. The idea is that the increase in $x_j$ increases the variance in pay from choosing the common project to such a high level that shareholders are better off by placing less weight on relative payoff.

To put it differently, $x_j$ has two effects over the variance of pay for bank $i$’s CEO: a variance effect (through $x_j^2$) and a covariance effect (linear in $x_j$). For low levels of $\psi$, $x_i$ and $x_j$ are small and the covariance effect dominates: an increase in $p_j$ leads to an increase in $x_j$ and because $x_i$ and $x_j$ are strategic complements, leads to an increase in $x_i$; the covariance effect is stronger and shareholders of bank $i$ increase $p_i$. For high levels of $\psi$, $x_i$ and $x_j$ are large and the variance effect dominates: an increase in $p_j$ increases $x_j$ and $x_i$, but $p_i$ decreases so as to reduce variance through the reduction in $x$ via the hedging demand.

5. Leverage

Up to now we assumed that, in addition to effort, the bank manager’s choice is limited to the allocation of $1 across two different assets. This precludes the possibility of leverage. By contrast, in this section we assume that the bank’s assets, $x_{ci} + x_{si}$, may be greater than the bank’s equity, which we continue to assume is fixed at $1$.

Introducing leverage shows that some of the intuitions presented earlier are remarkably robust; it also brings new ideas to the fore. Accordingly, in this section we focus primarily on differences with respect to the previous analysis. Assuming that the bank is able to borrow at the risk-free rate $r_b$, the bank’s return is now given by

$$r_i = e_i + x_{ci} \tilde{c}_i + x_{si} \tilde{s}_i + (1 - x_{ci} - x_{si}) r_b$$

We can then write

$$r_i = e_i + x_{ci} (\tilde{c}_i - r_b) + x_{si} (\tilde{s}_i - r_b) + r_b$$

or, defining $c_i = \tilde{c}_i - r_b$, $s_i = \tilde{s}_i - r_b$,

$$r_i = e_i + x_{ci} c_i + x_{si} s_i + r_b$$

(14)

Asset allocations are constrained by $x_{ci}, x_{si} > 0$. Leverage occurs when $x_{ci} + x_{si} > 1$. Below we provide a necessary and sufficient condition for positive leverage.

For simplicity, we maintain the assumptions that $\mu_i = \mu_c = \mu$, where $\mu$ is the expected value of $c_i$ and $s_i$; and that $\sigma_i = \sigma_c = \sigma = 1$. These assumptions allow us to focus on the strategic motives leading bank managers to choose a given portfolio (that is, motives different from each asset’s intrinsic value). Finally, we continue to assume that $\psi$ measures the correlation between the banks’ common project returns.

**Leverage ratios and balance sheet.** As mentioned earlier, our setup assumes that the bank has $1 of equity to invest. In the benchmark model (without leverage) the bank’s assets are given by $x + (1 - x) = 1$. With leverage, however, assets equal
equity plus debt, and so total assets can be larger than equity. Specifically, assets equals $x_c + x_s$, whereas leverage equals $(x_c + x_s) -$ $1 > 0$ (a negative number is the bank holds cash or a safe asset).

In this more general framework, the dollar amounts invested in the common and idiosyncratic projects ($c$ and $s$) can no longer also be seen as percentages of the value of equity, as in the benchmark model. Instead, we now express portfolio choices as percentages of total assets, $x_c + x_s$:

$$z \equiv x_c + x_s$$
$$x \equiv x_c/z$$
$$1 - x = x_s/z$$

The return

$$r_i = e_i + x_c \tilde{c}_i + x_s \tilde{s}_i + (1 - x_c - x_s) r_b$$

should therefore be interpreted as the return on equity, since $e_i + x_c \tilde{c}_i + x_s \tilde{s}_i$ is now the return on assets,

$$\frac{(x_c + x_s) - 1}{1} = z - 1 \equiv l$$

is now the debt/equity ratio (as well as the degree of leverage), and $r_b$ the return on debt.

**Compensation.** Similarly to (3), bank $i$ manager’s compensation is given by

$$w_i = k_i + a_i r_i - b_i r_j$$
$$= k_i + a_i (e_i + x_c c_i + x_s s_i + r_b) - b_i (e_j + x_cj c_j + x_sj s_j + r_b)$$
$$= k_i + a_i e_i - b_i e_j + a_i x_c ci - b_i x_cj c_j + a_i x_s si - b_i x_sj s_j + a_i r_b - b_i r_b$$

Similarly to (4)–(5), mean and variance of bank manager’s pay are given by

$$\mathbb{E}(w_i) = k_i + a_i e_i - b_i e_j + (a_i (x_c + x_s) - b_i (x_cj + x_sj)) \mu + (a_i - b_i) r_b$$  \hspace{1cm} (15)
$$\mathbb{V}(w_i) = a_i^2 x_c^2 + b_i^2 x_c^2 + 2 a_i b_i x_c x_cj \psi + a_i^2 x_s^2 + b_i^2 x_s^2$$  \hspace{1cm} (16)

**Leverage and portfolio composition.** Similarly to (6), the CEO’s utility maximization problem is now equivalent to

$$\max_{e_i, x_c, x_s} \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2$$

Similarly to (7), the first-order condition with respect to $e_i$ leads to

$$\hat{e}_i = a_i / \gamma_i$$

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Similarly to (9), the first-order condition with respect to $x_{ci}$ implies

$$x^*_{ci} = \frac{\mu + \psi b_i x_{cj}}{a_i}$$

(17)

The first-order condition with respect to $x_{si}$, in turn, implies

$$x^*_{si} = \frac{\mu}{a_i}$$

(18)

Notice that the strategic complementarity across banks is limited to investments in the common asset, $x_{ci}$. This may suggest that portfolio composition is different in a world with leverage. However, our first result shows that, as a function of the degree of RPE, portfolio composition is the same with or without leverage.

**Proposition 7.** In a symmetric equilibrium and for given RPE ratios $p_i \equiv b_i/a_i$, portfolio composition of assets $x_i$ are invariant with respect to the degree of leverage.

Before, we forced the level of leverage to be zero, that is, we forced total assets to add up to $1$. The next result characterizes the endogenous value of leverage chosen by bank managers if they have the freedom to do so.

**Proposition 8.** In a symmetric equilibrium ($a_1 = a_2 = a, b_1 = b_2 = b$), bank leverage $l$ is given by

$$l = x_c + x_s - 1 = \frac{\mu}{a} \frac{2 - \psi p}{1 - \psi p} - 1$$

For a given $p$, $l$ is decreasing in $a$; conversely, for a given $a$, $l$ is increasing in $p$.

Intuitively, an increase in incentive pay, $a_i$, leads to a greater variance in CEO pay. The latter optimally adjusts to such an increase by lowering investment levels.\(^8\) Conversely, an increase in RPE, as measured by $p$, leads to lower variance in CEO pay, for the reasons described earlier. The CEO optimally adjusts to such a decrease in variance by increasing investments levels.

Solving the equation in Proposition 8, we conclude that $l > 0$ if and only if

$$\mu > a (1 - \psi p)/(2 - \psi p)$$

One interpretation of this inequality is that, if expected returns from investment are sufficiently high with respect to the compensation parameters $a, p$ and the correlation

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\(^8\) An increase in $a_i$ also leads to a higher expected value of incentive pay, which in of itself would lead to higher investment levels; but the variance effect dominates.
coefficient $\psi$, then the CEO optimally chooses a positive degree of leverage. Alternatively, if $a$ is sufficiently low or $p$ is sufficiently high with respect to the value of $\mu$, then the CEO optimally chooses a positive degree of leverage.$^9$

We also note that RPE has an effect on systemic risk through two different channels. First, Proposition 3 states that an increase in $b$ leads to a portfolio composition that places greater weight on common projects; and Proposition 7 states that this effect is invariant with respect to the degree of leverage. Second, Proposition 8 shows that an increase in $b$ leads to an increase in the degree of leverage; and, for a given composition of CEO portfolios, an increase in leverage amplifies the systemic risk effect of CEO portfolio choices.

We conclude with a result that corresponds to Proposition 5 in the model without leverage. It does not provide a characterization as complete as that of Proposition 5, but shows that (a) RPE takes place in equilibrium; and (b) the degree of RPE is increasing in the degree of correlation across common projects, $\psi$.

**Proposition 9.** There exists a $\psi' > 0$ such that, if $0 < \psi < \psi'$, then in a symmetric equilibrium $p > 0$, $dp/d\psi > 0$, and $dx/d\psi > 0$. Moreover, the equilibrium value of $p$ is higher than in a model with no leverage.

Leverage increases systemic risk through two distinct channels: (i) holding portfolio composition constant, it increases systemic risk because it amplifies banks’ equity returns (this is the standard channel linking leverage to equity risk); (ii) levered banks feature a higher level of RPE and thus invest more in the correlated project. This second channel is unique to our paper.

6. Public Policy

At different banking jurisdictions, the recent regulatory trend has been to fix criteria for the design of pay structures that meet the international principles and standards issued by the Financial Stability Board in 2009 (FSB Principles for Sound Compensation Practice, 2009). These standards were formulated at a sufficient level of abstraction so as to allow for the smoothing of conflicts among members countries and insert flexibility in implementation. For example, with respect to the structure

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9. Note that there no costs of financial distress nor limited liability if the bank cannot pay its borrowed capital. If there are costs of financial distress, then bank shareholders have an interest in committing ex-ante — that is, when the borrowed capital is raised — to a low level of risk, for those costs are internalized by them at that time. Hence, financial distress costs should lead to lower RPE, lower leverage, a smaller allocation bias to the common project and lower systemic risk.

Limited liability, whether for CEOs or shareholders, gives an incentive to increase the bank’s risk level, which is accomplished by concentrating investment in a single project. If there is RPE, it is more advantageous to concentrate the investment in the common project, for that reduces the risk borne by the manager, thereby lowering the cost of managerial compensation and raising the level of effort deployed by the manager. In sum, limited liability is likely to strengthen the model’s results.
of pay, the FSB simply advocates the alignment of compensation with prudent risk taking, with the latter encompassing all types of risks.

In Europe the FSB standards were implemented through detailed rules enacted by primary legislation. The most important is the 4th Capital Requirements Directive (CRD IV, 2013), which states that variable compensation cannot exceed 100% of fixed pay, with at least 40% of it deferred for a minimum of 3 years. The European Banking Authority (EBA) subsequently issued detailed technical standards to clarify and interpret the rules enshrined in CRD IV. EBA takes a broad interpretation of variable compensation, including in it all compensation that is not contractually predetermined. It states that variable pay should be based on risk-adjusted performance and that the criteria to gauge performance may include measures of absolute performance as well as measures of relative performance vis-à-vis industry peers.

An extreme position is being taken by Israeli legislators, who have approved a cap on total pay of bank CEOs of 35 times the lowest salary paid by the firm, with a current value of cap at around 650,000 USD (Abudy and Saust, 2016). In contrast to Europe and Israel, the US has followed a regulatory approach based on the ex-post supervision of banks to check for consistency of FSB principles on sound compensation policies. Hence no specific quantitative limits on pay (such as caps on variable pay or floors on deferred pay) have been set.

In this section we use the model developed in the previous sections to remark on the strengths and weaknesses of some of these public policy measures and proposals. Our analysis suggests that they grossly omit the role that RPE plays in creating systemic risk, as shown in the previous sections.

CEO compensation includes several components: specifically, total pay is equal to fixed pay, $k_i$, plus variable pay (or pay for performance), $a_i r_i - b_i r_j$. Variable pay, in turn, is equal to incentive pay, $a_i r_i$, plus RPE pay, $-b_i r_j$. In what follows, we consider regulations that address each of these components of CEO compensation.

■ Caps on incentive pay. Consider first a cap in the form $a_i \leq \pi$, that is, an upper bound on the own-performance variable pay coefficient. The following result provides an irrelevance result that speaks to the ineffectiveness of incentive pay regulation.

**Proposition 10. In the model without leverage, a cap on incentive pay does not change the level of systemic risk. In the model with leverage, there exists $\psi' > 0$,**

10. Member states can set more stringent limits on variable pay. Member states may also allow shareholders to approve a higher maximum (up to 200%) by a supermajority vote (see article 94, (g) (ii)).
11. EBA also states that “relative measures could encourage excessive risk taking and need always to be supplemented by other metrics and controls” (Executive Summary, 44), but is unclear as to whether excessive risk refers to bank idiosyncratic risk or industry-wide risk.
12. An exception are financial institutions that were bailed out through TARP. The highest paid executives of these firms had their salaries capped at 500,000 USD while under the support of the US Treasury. Some authors observe that most firms accepting TARP funding did so before February of 2009, when the final pay restrictions were announced (Cadman and Carter, 2012).
such that, if $\psi < \psi'$, then imposing a binding cap $a_i \leq \pi$ results in an increase in leverage and in systemic risk.

Recall that in both models, the share of assets invested in the common project only depends on the ratio $p_i \equiv b_i/a_i$; and $p_i$ is thus a sufficient statistic for systemic risk. In the model without leverage, the variance of pay can be written as $V(w_i) = a_i^2 f(p_i, x_i^*(p_i), x_j)$, so the choice of $b_i$, which minimizes the variance of pay, is proportional to the choice of $a_i$. Thus, any active constraint on $a_i$ leads to a proportional change in $b_i$ that keeps $p_i$ constant and systemic risk unchanged. In the model with leverage, an active constraint capping the value of $a_i$ leads to a change in $b_i$ that is less than proportional, and $p_i$ increases. Intuitively, a lower $a_i$ leads to an increase in leverage, for fixed $p$ (see Proposition 8). The additional resources are used in both risky projects, but because of the benefits of RPE, $p$ increases to induce the manager to allocate relatively more to the common project. Thus, a cap on $a_i$ leads to an increase in systemic risk. Conversely, in the optimal contract high equity incentives also serve to limit risk-taking in order to limit the volatility of pay. It therefore acts as a constraint on leverage. Evidence on the negative association between equity incentives in banks and leverage can be found in John and Qian (2003).

Strictly speaking the actual proposal in CRD IV is not to cap $a_i$, but rather to cap variable play at 100% of fixed pay, that is $a_i r_i - b_i r_j \leq k_i$. This leads to a compensation level given by

$$w_i = k_i + \min \{a_i r_i - b_i r_j, k_i\}$$

The second component of pay is equivalent to the payout from a put option with the put’s underlying being $a_i r_i - b_i r_j$ and its strike price being $k_i$. Under this constraint, compensation is weakly increasing and concave on $a_i r_i - b_i r_j$. As the utility function is increasing and concave over $w_i$, the utility function remains increasing and concave over $a_i r_i - b_i r_j$. The shareholder therefore still cares about the negative effect that the volatility of $a_i r_i - b_i r_j$ has on the manager’s utility, and will try to use RPE to reduce that volatility. While the specific implications from a constraint that introduces a kink in compensation are hard to derive analytically in our setting, the mechanism in the previous sections should still apply, generating investments in the common project that are strategic complements and that increase in the amount of RPE.

While the effect of incentive-pay regulation does not seem to improve systemic risk in the model, it may actually have a strictly negative overall efficiency effect. A binding constraint that causes $a_i$ to be lower than the equilibrium outcome reduces the amount of effort by bank executives and lowers the value added of the financial industry.

We view our ineffectiveness result as an illustration of the argument put forth in Posner (2009, p. 297) that

Efforts to place legal limits on compensation are bound to fail, or to be defeated by loopholes, or to cause distortions in the executive labour market and in corporate behaviour.
More than a “loophole,” we argue that the compensation package already offers significant flexibility for shareholders to adjust to an artificial regulation; and that, as a result, no positive effect will take place in terms of systemic risk; rather, a negative effect (a “distortion”) may take place in “corporate behavior.”

The above discussion comes with a caveat. Our relatively simple model of banking competition is purposely simple and ignores potentially important features of the banking industry. Some of these may provide an independent justification for caps on variable pay. That possibility notwithstanding, our results suggest a fundamental weakness of the proposed measures: since RPE can be used to reduce the bank executive’s compensation risk, it can also be used to undo at least partly the intended risk-reduction goal of a cap on incentive pay.

Finally, we note that in the model without leverage a cap on variable pay reduces mean total compensation. To see this, recall that the individual participation constraint is given by

\[ \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i \epsilon_i^2 = u_i \]

In equilibrium

\[ \mathbb{V}(w_i) = a_i^2 x_i^2 + b_i^2 x_j^2 - 2 a_i b_i x_i x_j \psi + a_i^2 (1 - x_i)^2 + b_i^2 (1 - x_j)^2 \]

\[ = a_i^2 \left( x_i^2 + p_i^2 x_j^2 - 2 p_i x_i x_j \psi + (1 - x_i)^2 + p_i^2 (1 - x_j)^2 \right) \]

Because the term in curved brackets remains unchanged with the cap on \( a_i \) (recall that \( p \) and \( x \) are unchanged), \( \mathbb{V}(w_i) \) decreases with the cap on incentive pay (that is, \( \mathbb{V}(w_i) \) is increasing in \( a_i \)). Likewise \( e \) also decreases. Hence, mean total compensation decreases. Intuitively, the executive in the model is risk averse and cares about volatility. If she faces lower volatility, she does not require as much total pay. This result, for the model without leverage, contrasts with some arguments that mean total pay will not decrease (e.g., Murphy, 2013). In the model with leverage, a cap on \( a \) may result in an increase in leverage that increases volatility of total pay, in which case the executive requires greater compensation.

It is reasonable to think of imposing caps on the component of pay for peer performance, \( b \), since that’s what’s causing the bias towards the common project and the increase in systemic risk. In fact, we can show that in both models (with and without leverage) an active cap on \( b \) leads to a lower \( p \). In the model without leverage, this translates into lower investment in the common project and lower systemic risk, since the benefit of hedging is now lower for the executive. For the model with leverage it is not possible to sign the change in investment in the common project, since a lower \( b \) also leads to a lower \( a \) that pushes leverage up.\(^{13}\)

\[ \textbf{Caps on total pay.} \] To analyze the implications of a cap on total pay, we re-solve

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\(^{13}\) In practice, firms may chose to implement RPE in an implicit fashion by appropriately adjusting fixed pay over time. Such tactics would make any regulation over RPE hard to enforce.
the shareholders problem, (11), imposing an additional constraint on average pay:\textsuperscript{14}

**Proposition 11.** Consider a cap on total pay: $\mathbb{E}(w_i) \leq v$, where $v > 0$. In the model without leverage, the equilibrium level of systemic risk is unchanged. In the model with leverage, there exists a $\psi' > 0$ such that, if $\psi < \psi'$, then the level of leverage and of systemic risk increase.

To understand the intuition for this result, recall that, at the shareholder’s optimum, bank managers are held to their outside option:

$$\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - d_i(e_i) = u_i$$

A regulatory cap on $\mathbb{E}(w_i)$ must be compensated by a variation in $\mathbb{V}(w_i)$ or $d_i(e_i)$. How do changes in incentive pay $a_i$ change these components of bank CEO utility? In the proof, we show that $d\mathbb{V}(w_i)/da_i = 0$. The idea is that an increase in incentive pay is compensated by a decrease in leverage so as to maintain total variance constant.\textsuperscript{15}

Given this, the only way to increase CEO utility is by reducing effort level, which can only be induced by a decrease in $a_i$. This reduction in incentive pay leads to an increase in leverage. Moreover, for a given level of $b_i$, this leads to an increase in $p_i = b_i/a_i$, the index of RPE that determines systemic risk. (In the proof we show that changes in $b_i$ do not compensate for the change in $a_i$.)

In the model, the cap on total pay lowers incentives and effort, but does not change the nature of the moral hazard problem and hence the link between the two. This is consistent with Fahlenbrach and Stulz (2011) who show that there is no evidence that the relation between bank performance and CEO incentives is different for banks that received TARP money (and thus had a salary cap) and banks that did not.

In the Squam Lake Report (French et al., 2010), the authors recommend governments not to regulate the level of pay, partly due to the lack of evidence linking level of pay and risk-taking, and partly due to unintended consequences of regulating the level of pay, such as affecting the value added of the financial industry. Proposition 11 provides some support for this fear, to which we add the danger of further increasing leverage.

Strictly speaking the cap on total pay is not on ex-ante pay but on ex-post pay. The cap thus turns the pay of the executive into a short put option. Like the cap on incentive pay discussed above, the utility function remains increasing and concave over $a_i r_i - b_i r_j$, implying that the mechanism in the previous sections still applies and that the equilibrium should still deliver a bias towards the common project as well as RPE.

\textsuperscript{14} The specific point that caps in incentive pay can lead to lower effort in the banking industry has been made by several authors in different contexts (e.g., Bhagat et al., 2008; Murphy, 2009, 2013; Core and Guay, 2010). Other unintended consequences have also been discussed. For example, in Dittmann et al. (2011) average pay increases with caps on high powered incentives, in Hilscher et al. (2016) caps on ownership may lead to increased risk-taking, and in Asai (2016) caps on bonus may lead to more underinvestment.

\textsuperscript{15} The cross-partial derivative of $\mathbb{V}(w_i)$ with respect to $a_i$ and $x_{ci}$ is positive, thus an increase in one leads to an decrease in the other.
Caps on incentives or on total pay may work in the wrong way by increasing leverage. The next subsection discusses a more traditional “macro prudential” constraint that is more effective at curbing systemic risk in this model.

**Caps on leverage.** Consider now a cap on leverage, that is, \( l \equiv x_{ci} + x_{si} - 1 < L \).

What effect does this have on CEO choices and shareholder choices?

**Proposition 12.** Consider a cap on leverage: \( l \leq L \). If the cap is binding, then a decrease in \( L \) leads to (a) no change in the RPE ratio \( p \); (b) no change in portfolio composition \( x \); (c) an increase in variable pay (both \( a \) and \( b \)); (d) a decrease in systemic risk.

A direct effect of a decrease in leverage is to decrease CEO risk. Given this, shareholders optimally react by increasing the risk level of CEO compensation by increasing both \( a \) and \( b \). As \( a \) increases, CEO effort increases and so does productivity. A cap on leverage reduces systemic risk by reducing the traditional amplification effect of leverage on equity returns.

**Deferred pay.** The Financial Stability Board and the CRD IV call for performance to be evaluated over a multi-year period so as to

Ensure that the assessment process is based on longer-term performance and that the actual payment of performance-based components of remuneration is spread over a period which takes account of the underlying business cycle of the credit institution and its business risks (Article 94 of CRD IV).

In accordance with FSB recommendations, the CRD calls for deferments of 40%–60% of variable pay depending on the size of the pay for at least three years. Deferment periods are also being pursued by the UK’s Prudential Regulation Authority and the Financial Conduct Authority, arguing specifically that these are preferred to caps on incentive pay (see also French et al., 2010).

Our model can only be used to assess one of the potential benefits from deferred pay, perhaps not the most relevant one. By making a multi-year assessment, deferred pay excludes elements of business risk that are unrelated to managerial effort. In the limit when performance is measured over an infinite number of periods, there is no uncertainty in the effort-performance relation. In terms of our model, this would correspond to a decrease in the random component of performance to zero (in the limit).\(^{16}\)

\(^{16}\) Hoffmann et al. (2016) also model deferred pay with the benefit of more informative performance signals. Other models of deferred pay propose that it can allow the agent to achieve inter temporal risk sharing, but also, in combination with time varying vesting, to minimize short-termism (Edmans et al., 2012; Lambert, 1983; and Rogerson, 1985). Kolm et al. (2014) show that deferred pay can help limit excessive risk taking caused by risk shifting incentives when combined with a cap on the maximum bonus.
Even this “small” benefit of deferred pay already implies that RPE pay would cease to play a role as a way to reduce CEO risk. As such, we might say that deferred pay and RPE pay are substitutes. This is true too of other types of RPE as in Holmstrom (1979). In this sense, proposals that call for the consideration of more and varied metrics, financial and non-financial, to evaluate executive performance (see CRD IV Article 94(a)), can also act as deferred pay does, so long as they increase the precision with which contracted performance is measured.

However, it is not clear whether deferred pay, as a substitute for RPE, will lead to an industry equilibrium with lower systemic risk. For example, if the expected return on bank-specific projects is lower than that on common projects — even if infinitesimally so —, then the reduction of noise in the effort-performance relationship brought about by deferred pay will reduce the importance of risk diversification, thereby causing banks to load on common projects (as the high expected return alternative). Moreover, the dissipation of noise in the effort-performance relationship resulting from extending the number of periods of performance assessment may occur at a faster rate for common projects than for bank-specific projects, tilting the asset allocation of banks toward common projects too. That would occur, for example, if the noise in the effort-performance relationship associated with common projects features a lower degree of serial correlation than that associated with bank-specific projects.

To conclude this section, we should note that our policy analysis assumes that outside opportunities, denoted in our model by $u_i$, do not change with the proposed policy actions that we consider. However, some commentators (e.g., Murphy, 2013) argue that by lowering the level and structure of pay, pay restrictions reduce the attractiveness of senior management positions in the banking industry vis-à-vis other sectors of activity, decreasing the talent pool and reducing the long-term ability of the financial industry to generate value added for the rest of the economy.

7. Conclusion

Our main point is that, under RPE pay, risk-averse bank CEOs are likely to invest in common projects as a means to reduce the variance in pay. Anticipating such behavior, shareholders have an incentive to offer RPE as a means to reduce the expected value of CEO compensation required to satisfy the CEO’s participation constraint.

In other words, we uncover four sources of strategic complementarity: (a) under RPE pay, the more a CEO invests in a correlated project, the more the rival CEO wants to do the same; (b) the more a bank shareholder offers RPE pay, the more the rival bank’s shareholder wants to do the same; and (c) the more CEOs invest in correlated projects, the more shareholders want to increase the extent of RPE pay and vice-versa. Finally, (d) leverage adds another incentive to engage in RPE.

We derived a number of public policy implications of these results. One additional area that might be worth examining is international spillover effects. Suppose that
two banks in two different countries (e.g., Spain and Belgium) compete in the same market; and suppose that one of the countries (e.g., Belgium) enacts regulation that effectively reduces the level of investment in common assets. Even if the other country (Spain, in our example) does not impose a regulatory restriction on its banks, strategic complementarity leads the latter to decrease their investment in common assets, in tandem with Belgium banks. In addition, global banks, and large banks, have an incentive to use more RPE and they are also those that contribute most to systemic risk. When a global bank occupies a central position and is connected with many regional banks—each of which is connected to only other regional banks—the regional banks may do little RPE, whereas the larger bank may do a lot, giving rise to a buildup of systemic risk at the global bank.

We have assumed that project returns and variances are exogenous. If the price of the correlated investment project goes up and its expected return goes down for the same level of variance as more money is put into it, acting like decreasing returns, the incentive to take the correlated project would be attenuated and so would our mechanism. On the other hand, if by taking on similar strategies, the variance of the correlated project goes up (see for example Basak and Pavlova (2013)), then there would be an added incentive for more RPE and hence more investment in the correlated project possibly leading to a positive feedback loop. These mechanisms deserve further attention and are left for future research.
Appendix

Proof of Proposition 1: The proof follows by direct implication of (9). ■

Proof of Proposition 2: Proposition 1 implies that $x_i$ and $x_j$ are strategic complements. Moreover, from (9) and the assumptions that $b_i > 0$ and $a_i \geq b_i$
\[
\frac{dx_i}{dx_j} = \frac{b_i x_j}{2a_i} < \frac{b_i x_j}{a_i} \leq x_j \leq 1
\]
It follows that the reaction curves have a slope of strictly less than 1, which implies there exists a unique equilibrium. ■

Proof of Proposition 3: From (10), the variance of industry returns is given by
\[
\mathbb{V}(R) = x_1^2 + x_2^2 + 2 \psi x_1 x_2 + (1 - x_1)^2 + (1 - x_2)^2
\]
It follows that
\[
\frac{d\mathbb{V}(R)}{db_i} = 2 (\hat{x}_i + \psi \hat{x}_j - (1 - \hat{x}_i)) \frac{d\hat{x}_i}{db_i} + 2 (\hat{x}_j + \psi \hat{x}_i - (1 - \hat{x}_j)) \frac{d\hat{x}_j}{db_i}
\]
Substituting (9) for $x_i, x_j$, and simplifying, we get
\[
\frac{d\mathbb{V}(R)}{db_i} = 2 \left( \hat{x}_j + \frac{b_i \hat{x}_j}{a_i} \right) \frac{d\hat{x}_i}{db_i} + 2 \left( \hat{x}_i + \frac{b_j \hat{x}_i}{a_i} \right) \frac{d\hat{x}_j}{db_i}
\]
The terms in brackets are positive; and by Proposition 2 $d\hat{x}_k/db_i > 0$, $k = i, j$. It follows that $d\mathbb{V}(R)/db_i > 0$. ■

Proof of Proposition 4: At the optimum, the first constraint in (11) holds as an equality (and determines the value of $k_i$). Moreover $\mathbb{E}(r_i) = e_i$. The maximization problem is therefore equivalent to
\[
\max_{a_i, b_i} \quad e_i - \frac{1}{2} \gamma e_i^2 - \frac{1}{2} \mathbb{V}(w(x_i, x_j))
\]
s.t. \[
e_i = \hat{e}_i(a_i, b_i)
\]
\[
x_i = x_i^*(x_j; a_i, b_i)
\]
\[
x_j = x_j^*(x_i; \hat{a}_i, \hat{b}_i)
\]
or simply
\[
\max_{a_i, b_i} \quad \hat{e}_i - \frac{1}{2} \gamma \hat{e}_i^2 - \frac{1}{2} \mathbb{V}(w_i(x_i^*, x_j^*))
\]
where, for simplicity, we omit the arguments of $\hat{e}_i$, $x_i^*$ and $x_j^*$.
Consider the first-order condition with respect to $b_i$. From (7), $\hat{e}_i$ is not a function of $b_i$ or $x_i$. We thus focus on the partial derivative of $\mathbb{V}(w_i)$ with respect to $b_i$ as well as the effects through changes in $x_i$. 

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From (4) we see that \( \partial E(w_i)/\partial x_i = 0 \). It follows that the first-order condition for (6) that corresponds to \( x_i \) is equivalent to \( dV(w_i)/dx_i = 0 \). Given our assumption that bank \( i \)'s compensation contract is not observed by bank \( j \)'s CEO, it follows that \( dx_j^i/db_i = 0 \). In sum, the effects through CEO portfolio choices are zero. It follows that the first-order condition with respect to \( b_i \) is simply given by

\[
\frac{dV(w_i)}{db_i} = \frac{\partial V(w_i)}{\partial b_i} = 0
\]

From (5), this first-order condition is given by

\[
(a_i x_i \psi - b_i x_j) x_j - b_i (1 - x_j)^2 = 0
\]

which leads to

\[
b_i = \frac{\psi a_i x_i x_j}{x_j^2 + (1 - x_j)^2}
\]  

(20)

By the same argument as before, when computing the first-order condition with respect to \( a_i \) we can ignore the indirect effects through \( x_i \) and \( x_j \). We thus have

\[
(1 - \gamma_i e_i) \frac{de_i}{da_i} - \frac{1}{2} \frac{\partial V(w_i)}{\partial a_i} = 0
\]  

(21)

From (7), \( e_i = a_i/\gamma_i \) and \( de_i/da_i = 1/\gamma_i \). From (5)

\[
\frac{\partial V(w_i)}{\partial a_i} = 2 x_i (a_i x_i - \psi b_i x_j) + 2 a_i (1 - x_i)^2
\]

Substituting the above equalities into (21) and simplifying, the first-order condition with respect to \( a_i \) is given by

\[
\frac{1 - a_i}{\gamma_i} - x_i (a_i x_i - b_i x_j \psi) - a_i (1 - x_i)^2 = 0
\]

Solving for \( a_i \), we get

\[
a_i = \frac{1 + \gamma_i \psi b_i x_i x_j}{1 + \gamma_i x_i^2 + \gamma_i (1 - x_i)^2}
\]  

(22)

Finally, (20) and (22) imply that \( a_i, b_i > 0 \) for \( x_i, x_j > 0 \). ■

**Proof of Proposition 5:** Symmetry implies that \( x_i = x_j = x \) and \( p_i = p_j = p \), which in turn implies that (9) turns into

\[
x = \frac{1}{2} (1 + \psi p x)
\]

Solving for \( p \) we get

\[
p = \frac{2 x - 1}{\psi x}
\]  

(23)
The first-order condition with respect to the relative-performance parameter $b_i$ is given by
\[
\frac{\partial V(w_i)}{\partial b_i} = 2 b_i x_j^2 - 2 \psi a_i x_i x_j + 2 b_i (1 - x_j)^2 = 0.
\]
At a symmetric equilibrium, this becomes
\[
p = \frac{\psi x^2}{x^2 + (1 - x)^2}
\]
(24)

Define
\[
y \equiv \frac{1 - x}{x}
\]
(Note that $x$ is strictly decreasing in $y$ and that $x \in (\frac{1}{2}, 1)$ implies that $y \in (0, 1)$.) Given this change in variable, (23) and (24) may be re-written as
\[
\frac{1}{p} = \frac{\psi}{1 - y}
\]
\[
\frac{1}{p} = \frac{1 + y^2}{\psi}
\]
(Note that either equation implies that $p$ is strictly decreasing in $y$.) Together, these equations imply
\[
(1 - y) (1 + y^2) = \psi^2
\]
(25)

Computation establishes that (25) has two imaginary roots and a real root. Setting $\psi = 0$, the real root is $y = 1$, whereas setting $\psi = 1$ we get $y = 0$. Moreover, the derivative of the left-hand side with respect to $y$ is given by $1 + y (2 - 3 y)$, which is strictly positive for $y \in (0, 1)$, implying (by the implicit function theorem) that $y$ is decreasing in $\psi$. Since $p$ and $x$ are increasing in $y$, it follows that $p$ and $x$ are strictly increasing in $\psi$. Finally, from (25),
\[
\frac{1 - y}{\psi} = \frac{\psi}{1 + y^2}
\]
It follows that
\[
\frac{1}{p} = \frac{\psi}{1 - y} = \frac{1 + y^2}{\psi} > \frac{1}{\psi}
\]
where we use the fact that $x \in (0, 1)$ and thus $y > 0$. It follows that $p < \psi$ for $\psi < 1$.

**Proof of Proposition 6:** The first-order condition with respect to $b_i$ implies:
\[
(x_i \psi - p_i x_j) x_j - p_i (1 - x_j)^2 = 0
\]
Solving (12) for $x_i$, we get
\[
\hat{x}_i = \frac{2 + \psi p_i}{4 - \psi^2 p_i p_j}
\]
and

\[ 1 - \hat{x}_i = \frac{2 - \psi p_i (1 + \psi p_j)}{4 - \psi^2 p_i p_j} \]

Substituting into the first-order condition and simplifying,

\[ \Phi_i \equiv (2 + \psi p_i) (2 + \psi p_j) - p_i (2 + \psi p_j)^2 - p_i (2 - \psi p_i (1 + \psi p_j))^2 = 0 \quad (26) \]

Differentiating with respect to \( p_i \), we get

\[ \frac{\partial \Phi_i}{\partial p_i} = \psi (2 + \psi p_j) - (2 + \psi p_j)^2 - (2 - \psi p_i (1 + \psi p_j))^2 + 2 p_i (2 - \psi p_i (1 + \psi p_j)) \psi (1 + \psi p_j) \]

At a symmetric equilibrium, \( p_i = p_j = p \). Moreover, Proposition 5 implies that \( p = 0 \) if \( \psi = 0 \) and \( p = 1 \) if \( \psi = 1 \). Therefore

\[ \left. \frac{\partial \Phi_i}{\partial p_i} \right|_{\psi = 0} = -8, \quad \left. \frac{\partial \Phi_i}{\partial p_i} \right|_{\psi = 1} = -6 \]

The implicit-function theorem implies that, in the neighborhoods of \( \psi = 0 \) and \( \psi = 1 \), the sign of the slope of \( B_i(p_j) \), shareholder \( i \)'s best-response mapping, is the same as the sign of \( \frac{\partial \Phi_i}{\partial p_j} \). Differentiating (26), we get

\[ \frac{\partial \Phi_i}{\partial p_j} = \psi (2 + \psi p_i) - 2 \psi p_i (2 + \psi p_j) + 2 \psi^2 p_i^2 (2 - \psi p_i (1 + \psi p_j)) \]

which implies

\[ \left. \frac{\partial \Phi_i}{\partial p_j} \right|_{\psi = 0} = 2 \psi, \quad \left. \frac{\partial \Phi_i}{\partial p_j} \right|_{\psi = 1} = -3 \]

The result then follows by continuity. (Notice in particular that, at \( \psi = 0 \), \( \frac{\partial \Phi_i}{\partial p_j} = 0 \), but in the right neighborhood where \( \psi > 0 \) we have \( \frac{\partial \Phi_i}{\partial p_j} > 0 \).)

**Proof of Proposition 7:** From (17) and (18), we derive the value of \( x \), the relative weight of common assets in total assets:

\[ x^*_i = \frac{1}{2} + \frac{\psi b_i x_{cj}}{2 a_i} \]

It follows that, in a symmetric equilibrium, this is the same as (9).

**Proof of Proposition 8:** In a symmetric equilibrium, (17)–(18) imply

\[ x_c = \frac{\mu + \psi b x_c}{a} \]
\[ x_s = \frac{\mu}{a} \]
Adding up and simplifying, we get

\[ z = x_c + x_s = \frac{\mu 2 - \psi p}{a 1 - \psi p} \]  

(27)

The result follows from taking partial derivatives. ■

**Proof of Proposition 9:** Bank \( i \)'s shareholders solve

\[
\max_{k_i, a_i, b_i} \mathbb{E}(r_i - w_i)
\]

s.t. \[
\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2 \geq u_i
\]

\[
e_i = e_i^*(a_i)
\]

\[
x_i = x_i^*(a_i, b_i; x_j)
\]

Substituting the IR constraint, this becomes

\[
\max \{ \mathbb{E}(r_i) - u_i - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2 \}
\]

s.t. \[
\mathbb{E}(r_i) = e_i^* + \mu x_{ci}^* + \mu x_{si}^* + r_b
\]

\[
\mathbb{V}(w_i) = a_i^2 x_{ci}^2 + b_i^2 x_{cj}^2 - 2 \psi a_i b_i x_{ci} x_{cj} + a_i^2 x_{si}^2 + b_i^2 x_{sj}^2
\]

\[
a_i x_{ci}^* = \mu + \psi b_i x_{cj}
\]

\[
x_{si}^* = \frac{\mu}{a_i}
\]

\[
e_i^* = \frac{a_i}{\gamma_i}
\]

The first-order condition with respect to \( b_i \) is given by

\[
-\frac{1}{2} \frac{d \mathbb{V}(w_i)}{db_i} + \frac{\partial \left( \mu x_{ci}^* - \frac{1}{2} \mathbb{V}(w_i) \right)}{\partial x_{ci}^*} \frac{\partial x_{ci}^*}{db_i} = 0
\]

Note that

\[
\frac{\partial x_{ci}^*}{\partial b_i} = \frac{\psi x_{cj}}{a_i}
\]

\[
\frac{\partial \mathbb{V}(w_i)}{\partial x_{ci}^*} = 2 \left( a_i x_{ci}^* - \psi b_i x_{cj} \right) a_i
\]

\[
\frac{\partial \mathbb{V}(w_i)}{\partial b_i} = -2 \left( \psi a_i x_{ci}^* - b_i x_{cj} \right) x_{cj} + 2 b_i x_{sj}^2
\]

The first-order condition thus becomes:

\[
-\frac{1}{2} \left( -2 \left( \psi a_i x_{ci}^* - b_i x_{cj} \right) x_{cj} + 2 b_i x_{sj}^2 \right) + \left( \mu - \left( a_i x_{ci}^* - \psi b_i x_{cj} \right) a_i \right) \frac{\psi x_{cj}}{a_i} = 0
\]

Using the first-order condition with respect to \( x_{ci} \), the equilibrium values of \( x_{si} \) and of \( x_{cj} \), and simplifying, we get

\[
(\psi^2 - 1) b_i \left( x_{cj}^* \right)^2 + \frac{\psi \mu x_{cj}}{a_i} - b_i x_{sj}^2 = 0
\]
or simply
\[(\psi^2 - 1) b_i \left( \frac{a_i + \psi b_i}{a_j a_i - \psi^2 b_i b_j} \right)^2 + \frac{\psi}{a_i} \frac{a_i + \psi b_j}{a_j a_i - \psi^2 b_i b_j} - \frac{b_i}{a_j^2} = 0\]

Imposing symmetry (that is, \(a_i = a_j, b_i = b_j\)),
\[(\psi^2 - 1) b \left( \frac{1}{a - \psi b} \right)^2 + \frac{\psi}{a (a - \psi b)} - \frac{b}{a^2} = 0 \quad (28)\]

The first-order condition with respect to \(a_i\) is given by
\[\left(1 - \gamma_i e_i^*\right) \frac{d e_i^*}{d a_i} - \frac{1}{2} \frac{\partial V(w_i)}{\partial a_i} + \frac{\partial \left(\mu x_{ci}^* - \frac{1}{2} V(w_i)\right)}{\partial x_{ci}^*} \frac{\partial x_{ci}^*}{\partial a_i} + \frac{\partial \left(\mu x_{si}^* - \frac{1}{2} V(w_i)\right)}{\partial x_{si}^*} \frac{\partial x_{si}^*}{\partial a_i} = 0 \quad (29)\]

Note that
\[\frac{\partial x_{ci}^*}{\partial a_i} = -a_i^{-2}(\mu + \psi b_i x_{cj})\]
\[\frac{\partial x_{si}^*}{\partial a_i} = -\mu a_i^{-2}\]

Substituting these in (29); substituting the equilibrium values of \(x_{ci}, x_{si}\); and imposing symmetry (that is, \(\gamma_i = \gamma, x_{ci} = x_c, \text{ etc}\)), (29) becomes
\[\frac{1 - a}{\gamma} - \frac{\mu^2 (2 a - \psi b)}{a^2 (a - \psi b)} = 0 \quad (30)\]

Substituting \(p\) for \(b/a\) in (28) and (30), we get a system of equations defining the equilibrium values of \((a, p)\):
\[A \equiv (1 - a) \frac{a^2}{\gamma} - \mu^2 \frac{2 - \psi p}{1 - \psi p} = 0 \quad (31)\]
\[B \equiv (\psi^2 - 1) p \left( \frac{1}{1 - \psi p} \right)^2 + \frac{\psi}{(1 - \psi p) a} - p = 0 \quad (32)\]

If \(\psi = 0\), then \(b = p = 0\) and \(a = a_0\). Given this, we can take partial derivatives of \(A\) and get
\[\frac{\partial A}{\partial \psi} \bigg|_{\psi = 0} = \frac{\partial A}{\partial p} \bigg|_{\psi = 0} = 0\]

It follows by the implicit function theorem that
\[\frac{d a}{d \psi} \bigg|_{\psi = 0} = 0\]
We can therefore apply the implicit function theorem to (32) and get
\[
\frac{dp}{d\psi} \bigg|_{p=0, \psi=0} = -\frac{1/a_0}{-2} > 0
\]
It follows that for \( \psi \) greater than, but different from zero, \( p \) is positive and increasing in \( \psi \). By Propositions 5 and 7, the same is true of \( x \).

Consider now the model without leverage. Totally differentiating (24) at \( \psi = p = 0 \), we get
\[
dp = \frac{x^2}{x^2 + (1-x)^2} d\phi
\]
(Notice the derivative with respect to \( x \) multiplies \( \psi \).) Since \( x = \frac{1}{2} \) when \( \phi = 0 \), we get
\[
\frac{dp}{d\psi} \bigg|_{p=0, \psi=0} = \frac{1}{2}
\]
This is smaller than the corresponding derivative in the model with leverage if and only if \( a_0 < 1 \), where \( a_0 \) is the equilibrium value of \( a \) when \( \psi = 0 \). Substituting 0 for \( p \) and \( \psi \) in (30), we get
\[
(1-a) a^2 = 2 \mu^2 \gamma
\]
which implies that \( 0 < a_0 < 1 \).

**Proof of Proposition 10:** Consider first the model without leverage. Assume that the constraint on incentive pay is active, \( a_i = \overline{a} \), otherwise there would be no change in the game’s equilibrium outcome. Note that the equilibrium is still characterized by the solution \((p, x)\) that solves (24), because (24) results from the first-order condition for \( b_i \), which still holds with equality. Once the equilibrium value of \( p \) is determined, \( b_i \) (and \( b_j \)) can be appropriately adjusted for any given \( a_i \) (and \( a_j \)). Thus, a binding constraint on \( a \) affects the value of \( b \) but not the value of \( p \). It follows that portfolio choices \( x \) remain unaltered, keeping the level of systemic risk unchanged.

Consider now the model with leverage. Given that the constraint on \( a \) is binding, the equilibrium value of \( b \) is determined by (32) where the value of \( a \) is treated as an exogenous parameter (basically \( a = \overline{a} \)). Applying the implicit function theorem, we get
\[
\frac{dp}{da} \bigg|_{p=0, \psi=0} = -\frac{\partial B / \partial a}{\partial B / \partial p} \bigg|_{p=0, \psi=0} = -\frac{-\overline{\psi}}{\overline{a}^2} = \frac{\psi}{-2}
\]
This is zero at \( \psi = 0 \), but approaches zero from negative numbers. Hence, by continuity \( dp/da < 0 \) for low enough \( \psi \). With lower \( a \) and higher \( p \), \( x_c \) increases and so does \( x_s \). Leverage increases and so does systemic risk.
Proof of Proposition 11: Suppose that $E(w^*_i) > v$, where $w^*_i$ corresponds to the unconstrained solution. Then the cap matters, that is, $E(w^*_i) = v$. Consider first the model without leverage. Then (11) may be written as

$$\max_{a_i,b_i} e_i - v$$

subject to the participation constraint,

$$v - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i$$
as well as the constraint that $e_i$ and $x_i$ belong to the best-response mappings.

Notice that $b_i$ is not present in the objective function: from (7), $e_i$ is a function of $a_i$ but not $b_i$. It follows that the optimal $b_i$ maximizes the slack in the participation constraint. As shown in the proof of Proposition 4, this implies $\partial V(w_i)/\partial b_i = 0$, which in turn determines the value of $p_i = b_i/a_i$. It follows that the same value of $p_i$ obtains as in the problem without the cap on pay.

Consider now the model with leverage. The problem faced by shareholders is:

$$\max_{k_i,a_i,b_i} E(r_i - w_i)$$

subject to

$$E(w_i) - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i$$

and that $e_i$, $x_c$ and $x_s$ belong to the best-response mappings. Let $k_i$ be such that $E(w_i) = v$ and rewrite the problem as:

$$\max_{a_i,b_i} E(r_i) - v \quad (33)$$

subject to

$$v - \frac{1}{2} V(w_i) - \frac{1}{2} \gamma_i^{-1} a_i^2 \geq u_i \quad (34)$$

Let $a_i = f(b_i; v, x_{cj}, x_{sj})$ be the solution to (34), as an equality, with respect to $a_i$ (note that bank $i$’s shareholders take $x_{cj}$ and $x_{sj}$ as given). Also, recall that

$$E(r_i) = e^*_i + \mu x^*_{ci} + \mu x^*_{si} + r_b$$

Then we can re-write (49)–(34) as

$$\max_{b_i} \left\{ \gamma_i^{-1} f(b_i) + \mu \left( \frac{\mu}{f(b_i)} + \psi \frac{b_i}{f(b_i)} x_{cj} \right) + \frac{\mu^2}{f(b_i)} + r_b - v \right\} \quad (35)$$

In order to maximize (35) we must compute the derivative of $f(b_i)$, where $a_i = f(b_i; v, x_{cj}, x_{sj})$. Since this is derived from (34), which includes $V(w_i)$, we must compute the derivatives of $V$ with respect to $a_i, b_i$. 

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Partial derivatives of $V(w_i)$ with respect to $a_i$ and $b_i$. First we show that $dV(w_i)/da_i = 0$. Note that this derivative takes $x_{cj}$ and $x_{sj}$ as given because we’re working with the problem of bank $i$’s shareholders and assume that bank $i$’s contract is not observed by bank $j$’s CEO. Taking the derivative of (16) with respect to $a_i$, we get

$$
\frac{\partial V(w_i)}{\partial a_i} = 2 a_i x_{ci}^2 - 2 \psi b_i x_{ci} x_{cj} + 2 a_i x_{si}^2 - 2 a_i x_{ci} \frac{\mu + \psi b_i x_{cj}}{a_i^2} + 2 \psi a_i b_i x_{cj} \frac{\mu + \psi b_i x_{cj}}{a_i^2} - 2 a_i x_{si} \frac{\mu^2}{a_i^2} \\
= 2 a_i x_{ci}^2 - 2 \psi b_i x_{ci} x_{cj} + 2 a_i x_{si}^2 - 2 a_i x_{ci}^2 + 2 \psi b_i x_{cj} x_{ci} - 2 \mu x_{si} \\
= 2 a_i x_{si}^2 - 2 \mu x_{si} \\
= 2 a_i \frac{\mu^2}{a_i^2} - 2 \mu x_{si} \\
= 0
$$

where we substitute (18) for $x_{si}$. We next compute the value of $dV(w_i)/db_i$. Taking the derivative of (16) with respect to $b_i$, we get

$$
\frac{\partial V(w_i)}{\partial b_i} = 2 (1 - \psi^2) b_i x_{cj}^2 + 2 b_i x_{sj}^2 \\
= 2 (1 - \psi^2) b \left( \frac{\mu}{a - \psi b} \right)^2 + 2 b \left( \frac{\mu}{a} \right)^2
$$

where we substitute (17) and (18) for $x_{ci}$ and $x_{si}$.

We next use these derivatives, $\partial V(w_i)/\partial a_i$ and $\partial V(w_i)/\partial b_i$, evaluated at the equilibrium values, in the solution to (35). The first-order condition for is given by

$$
\frac{dE(r_i)}{db_i} + \frac{dE(r_i)}{da_i} \frac{df}{db_i} = 0
$$

or

$$
\psi \frac{\mu}{a_i} x_{cj} + \left( \gamma_i^{-1} - \mu \left( \frac{\mu}{a_i^2} + \psi \frac{b_i}{a_i^2} x_{cj} \right) \right) \frac{\mu^2}{a_i^2} \frac{df}{db} = 0 \quad (36)
$$

Applying the implicit function theorem to (34) as an equality, we get

$$
\frac{df}{db} = -\frac{1}{2} \frac{dV(w_i)}{da} + \gamma_i^{-1} a_i
$$

Substituting for $df/db$ in to (36) and simplifying we get

$$
\mu \psi x_{cj} - \left( 1 - 2 \gamma_i \frac{\mu^2}{a_i^2} - \mu \psi \gamma_i \frac{b_i}{a_i^2} x_{cj} \right) \frac{1}{2} \frac{dV(w_i)}{db} = 0 \quad (37)
$$
Dropping the bank indexes $i, j$; substituting the result for $dV(w)/db$ in (37); substituting $p$ for $b/a$; and simplifying, we get

$$
\psi - \left( 1 - \frac{\mu^2 2 - \psi p}{a^2 1 - \psi p} \right) \left( \frac{1 - \psi^2}{1 - \psi p} + 1 - \psi p \right) p = 0
$$

(38)

Ultimately, we want to derive an equilibrium expression including $p$ (endogenous variable) and $\psi$ (exogenous parameter). The above expression includes another endogenous variable, $a$. We have another equation from which the value of $a$ can be obtained: (34), written as an equality. This expression includes the term $V(w_i)$, which is given by (16). Imposing symmetry, this becomes

$$(a^2 + b^2) (x_c^2 + x_s^2) - 2 \psi a b x_c^2$$

Substituting (17)–(18) (with subscripts $i, j$ dropped) for $x_c$ and $x_s$, and simplifying, we get

$$
V(w_i) = \mu^2 \left( \frac{1 + p^2 - 2 \psi p}{(1 - \psi p)^2} + 1 + p^2 \right) \equiv g(p)
$$

(39)

Substituting for $V(w_i)$ in (34), written as an equality, and solving for $a^2$, we get

$$
a^2 = 2 \gamma (v - u) - \gamma g(p)
$$

(40)

Substituting (40) for $a^2$ in (38), we get

$$
0 = \Phi(p, v) \equiv \\
\psi - \left( 1 - \frac{\gamma \mu^2 (2 - \psi p)}{(2 \gamma (v - u) - \gamma g(p)) (1 - \psi p)} \right) \left( \frac{1 - \psi^2}{1 - \psi p} + 1 - \psi p \right) p
$$

(41)

We next compute this derivative at $\psi = 0$. Recall that $\psi = 0$ implies $b = p = 0$.

$$
\frac{\partial \Phi}{\partial p} \bigg|_{p=0, \psi=0} = -2 \left( 1 - \frac{\mu^2}{v - u - \mu^2} \right) < 0
$$

It follows from the implicit-function theorem that the sign of $dp/dv$ is the same as the sign of $\partial \Phi/\partial v$. From (41), we get

$$
\frac{\partial \Phi}{\partial v} \bigg|_{p=0, \psi=0} = -2 \left( \frac{\mu}{v - u - \mu^2} \right)^2 p
$$

Although this expression equals zero when $\psi = p = 0$, it converges to zero by means of a sequence of negative values as $\psi \to 0^+$. Therefore, there exists a $\psi' > 0$ such that, if $\psi < \psi'$, then $dp/dv < 0$.

Next we consider the effects of $v$ on leverage $z$. Taking total derivatives,

$$
\frac{dz}{dv} \bigg|_{p=0, \psi=0} = \frac{\partial z}{\partial a} \bigg|_{p=0, \psi=0} \frac{\partial a}{\partial v} \bigg|_{p=0, \psi=0} + \frac{\partial z}{\partial p} \bigg|_{p=0, \psi=0} \frac{\partial p}{\partial v} \bigg|_{p=0, \psi=0}
$$
From (27),
\[ \frac{\partial z}{\partial a} \bigg|_{\psi = 0} = \frac{-2 \mu}{\sigma^2} \]
\[ \frac{\partial z}{\partial p} \bigg|_{\psi = 0} = 0 \]

From (40), \( \partial a / \partial v > 0 \). It follows that \( dz / dv < 0 \).

**Proof of Proposition 12:** Bank managers solve
\[ \max_{e_i, x_{ci}, x_{si}} \mathbb{E}(w_i) - \frac{1}{2} \gamma_i e_i^2 \]
subject to
\[ x_{ci} + x_{si} \leq 1 + L \]
and where \( \mathbb{E}(w_i) \) and \( \mathbb{V}(w_i) \) are given by (15) and (16), respectively. Since \( x_{ci} \) and \( x_{si} \) enter additively in the leverage constraint, we have
\[ \frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial x_{ci}} = \frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial x_{si}} \quad (42) \]

Intuitively, if there is a constraint on the sum \( x_{ci} + x_{si} \), then the marginal utilities with respect to \( x_{ci} \) and \( x_{si} \) must be the same (zero if the constraint is not binding). From (15)–(16),
\[ \frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial x_{ci}} = \mu a_i - (a_i^2 x_{ci} - \psi a_i b_i x_{cj}) \]
\[ \frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial x_{si}} = \mu a_i - a_i^2 x_{si} \]

Given (42), we get
\[ \mu a_i - (a_i^2 x_{ci} - \psi a_i b_i x_{cj}) = \mu a_i - a_i^2 x_{si} \]
or simply
\[ x_{si} - x_{ci} = \psi \frac{b_i}{a_i} x_{cj} \quad (43) \]

If the leverage constraint is binding, then
\[ x_{ci} + x_{si} = 1 + L \quad (44) \]
Together, (43)–(44) imply
\[ x_{ci}^* = \frac{1}{2} (1 + L) + \frac{1}{2} \psi \frac{b_i}{a_i} x_{cj} \quad (45) \]
\[ x_{si}^* = \frac{1}{2} (1 + L) - \frac{1}{2} \psi \frac{b_i}{a_i} x_{cj} \quad (46) \]
Intuitively, portfolio allocation does not respond to \( \mu \) since the leverage constraint is binding. For future reference, notice that

\[
\frac{dx_{si}}{da_i} = -\frac{dx_{ci}}{da_i} \tag{47}
\]

\[
\frac{dx_{si}}{db_i} = -\frac{dx_{ci}}{db_i} \tag{48}
\]

The problem faced by shareholders is:

\[
\max_{k_i, a_i, b_i} \mathbb{E}(r_i - w_i)
\]

subject to

\[
\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - d_i(e_i) \geq u_i
\]

and that \( e_i, x_c \) and \( x_s \) belong to the best-response mappings. Let \( k_i \) be such that the constraint is exactly satisfied and rewrite the problem as:

\[
\max_{a_i, b_i} \mathbb{E}(r_i) - u_i - \frac{1}{2} \mathbb{V}(w_i) - d_i(e_i) \tag{49}
\]

where \( \mathbb{E}(w_i) \) and \( \mathbb{V}(w_i) \) are given by (15) and (16), respectively; and \( x_{ci}, x_{si} \) by (45) and (46), respectively. The first-order conditions with respect to \( a_i \) and \( b_i \) is given by

\[
\frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial a_i} + \frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial x_{ci}} \frac{dx_{ci}}{da_i} + \frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial x_{si}} \frac{dx_{si}}{da_i} = 0
\]

\[
\frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial b_i} + \frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial x_{ci}} \frac{dx_{ci}}{db_i} + \frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial x_{si}} \frac{dx_{si}}{db_i} = 0
\]

Given (42) and (47), this simplifies to

\[
\frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial a_i} = 0
\]

\[
\frac{\partial (\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i))}{\partial b_i} = 0
\]

From (15)–(16), we get

\[
(1 - a_i)/\gamma_i - \frac{1}{2} \left( 2a_i x_{ci}^2 - 2\psi b_i x_{ci} x_{cj} + 2a_i x_{si}^2 \right) = 0
\]

\[
-2(\psi a_i x_{ci} - b_i x_{cj}) x_{cj} + 2b_i x_{sj}^2 = 0
\]

In a symmetric equilibrium

\[
(1 - a)/\gamma - ((a - \psi b)x_c^2 + a x_s^2) = 0 \tag{50}
\]

\[
-(\psi a - b)x_c^2 + b x_s^2 = 0 \tag{51}
\]
Substituting $p$ for $b/a$ and solving (45)–(46) for the symmetric equilibrium, we get

\begin{align*}
    x_c &= \frac{1}{2 - \psi p} (1 + L) \\
    x_s &= \frac{1 - \psi p}{2 - \psi p} (1 + L)
\end{align*}

Substituting for $x_c$ and $x_s$ in (50)–(51) and simplifying,

\begin{align*}
    (1 - a)/\gamma - a \left( \frac{1 - \psi p}{2 - \psi p} \right) (1 + L)^2 &= 0 \\
    -(\psi - p) + p (1 - \psi p)^2 &= 0
\end{align*}

From the second equation, we see that $p$ is determined by $\psi$ and independent of $L$. Moreover, from the first equation,

\[ \frac{da}{dL} = -\frac{2a \left( \frac{1 - \psi p}{2 - \psi p} \right) (1 + L)}{\gamma^{-1} + \left( \frac{1 - \psi p}{2 - \psi p} \right) (1 + L)^2} < 0 \]

Finally, from (52)–(53), we get

\[ x = \frac{x_c}{x_c + x_s} = \frac{1}{2 - \psi p} \]

The result follows. ■
References


