

The equilibrium consequences of indexing

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Abstract

We study the consequences of indexing, i.e. committing to invest in risky assets only via the market portfolio. We extend the canonical rational expectations model (Grossman and Stiglitz, 1980) to allow for multiple assets and endowment shocks, and show that indexing imposes a negative externality on other uninformed agents. More indexing makes informed trading on the market more profitable, which decreases welfare by distorting risk-sharing.

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1 Introduction

The standard investment recommendation that academic financial economists offer to retail investors is to purchase a low-fee index fund. This recommendation is often justified by an appeal to the “average investor theorem,” which states that the average investor’s return must equal the market return.¹ As such, an individual investor can avoid trading at a disadvantage with better-informed investors by holding a market index fund.²

In this paper we ask the following simple but unanswered question: What are the equilibrium consequences of this standard recommendation to index? We ask this question using the canonical model of informed trade, namely a version of Grossman-Stiglitz-Hellwig in which “liquidity trades” are explicitly motivated as stemming from stochastic exposure to economic shocks that are correlated with the cash flows of traded assets. In order to evaluate the recommendation to index, we extend this model to a case with multiple (two) distinct assets.³ Investors in our model have heterogeneous abilities to acquire information about future asset payoffs. Our focus is on how indexing affects the welfare of investors with below-average information-acquisition abilities, who we refer to as retail investors.

Our main result is that as the fraction of retail investors who index rises, the utility of all retail investors who index falls. The fact that an individual retail investor suffers from indexing is a straightforward implication of the canonical model. In the canonical model, a low asset price may reflect either that informed investors have negative expectations

¹The empirical basis for this recommendation is mixed: see, in particular, Berk and van Binsbergen (2016).

²For example, Cochrane (2013) writes: “The average investor theorem is an important benchmark: The average investor must hold the value-weighted market portfolio. Alpha, relative to the market portfolio, is by definition a zero-sum game. For every investor who over-weights a security or invests in a fund that earns positive alpha, some other investor must underweight the same security and earn the same negative alpha. Collectively, we cannot even rebalance. And each of us can protect ourselves from being the negative-alpha mark with a simple strategy: hold the market portfolio, buy or sell only the portfolio in its entirety, and refuse to trade away from its weights, no matter what price is offered. If every uninformed trader followed this strategy, informed traders could never profit at our expense.”

³Note that in Admati (1985) and other previous analyses of multiple-asset economies, liquidity trades are entirely exogenous.

about the asset’s future cash flows, or that the aggregate exposure to economic shocks correlated with these cash flows is high, generating a high discount rate for these cash flows. Consequently, if a retail investor observes a low price for an asset, and has no exposure to economic shocks, he should take a long position in the asset, since its conditional expected return is high. That is, a retail investor can profit from buying “value” stocks, which an indexing strategy would preclude. Although this point is often overlooked, it is nonetheless a standard implication of the canonical model (see, e.g., Biais, Bossaerts, and Spatt, 2010).

In contrast, the result that indexing exerts a negative pecuniary externality on other retail investors who index is new to our paper. The intuition for this result is as follows. It is useful to first note that portfolio positions in a two-asset economy can be decomposed into a holding of a market index and a “spread” asset that is long one asset and short another. Indexing represents a decision not to trade in the spread asset. Consequently, indexing by retail investors reduces the amount of liquidity trading by retail investors in the spread asset. *Ceteris paribus*, this makes the price of the spread asset more informative about future cash flows, which in turn leads informed investors to acquire less information about the spread asset. Informed investors then substitute towards acquiring more information about the market asset. Even when trading the market asset, retail investors suffer a disadvantage relative to informed investors, since, conditional on the price, informed investors buy exactly when future cash flows are likely to be high, while retail investors do just the opposite. This disadvantage only grows when informed investors acquire more information, reducing the welfare of retail investors.

In addition to showing that, in equilibrium, indexing reduces the welfare of retail investors, our analysis produces a number of empirical implications. First, indexing reduces the market risk premium. Second, indexing leads to higher levels of “price efficiency,” as measured by the ability of prices to predict future cash flows. Third, indexing leads to a shift in trading strategies of informed investors, who shift from trades based on cross-sectional

“mispricing” to ones that focus instead on aggregate factors. These implications are broadly consistent with the recent empirical literature on hedge funds and ETFs (Fung, Hsieh, Naik, and Ramadorai, 2008; Sun, Wang, and Zheng, 2012; Ramadorai, 2013; Pedersen, 2015; Israeli, Lee, and Sridharan, 2015).

Our model essentially extends Admati (1985) by explicitly modelling endowments shocks, as in Diamond and Verrecchia (1981). While the retail sector in our model, a fraction of whom are indexers, are uninformed, we allow the informed agents to choose the precisions of signals, as in Verrecchia (1982), on both the aggregate market, or on the relative value of the two stocks, via the “spread” asset. We note that our indexing agents can still time the market, but they cannot tilt their portfolios towards one particular asset, or, using the lingo from Admati, Bhattacharya, Pfleiderer, and Ross (1986), they can do timing, but not selectivity. In contrast to Van Nieuwerburgh and Veldkamp (2009), who focus on information acquisition in multi-asset markets using “capacity” constraints defined over the whole variance-covariance matrix of signals’ errors, we explicitly model the signals that informed agents can receive, and allow the informed agents to choose the precision of each signal’s errors.

While with very different focus, Marín and Rahi (1999), Ganguli and Yang (2009) and Manzano and Vives (2011) are closely related. Like these papers, we also use endowment shocks as the modelling device that prevents prices from fully revealing the informed agents’ signals. The main reason for such a choice is two fold: (1) we want a setting where trading motives are explicitly modelled, (2) we can do welfare analysis without having to judge what to do with, say, Kyle (1985)’s noise traders. An important implication of this assumption is that even when the retail sector does not receive signals on assets payoffs, they do have private signals on aggregate endowment shocks, provided by their individual endowment realization. Relative to the afore-mentioned papers, we model an economy with multiple risk assets, and we explicitly characterize welfare (a step that these previous papers have

struggled with).

Our paper is also closely related to the recent literature on the rise of indexing investing. Stambaugh (2014) documents a large drop in individual equity ownership over the last three decades, coupled with a similar drop in the share of institutional money that is actively managed. Our paper shares with his model the fact that more indexing means “less noise” in financial markets, although the mechanisms are quite different: we explicitly model trading stemming from endowment shocks, whereas Stambaugh (2014)’s noise is closer to the behaviorally biased agents of Black (1986).

The rest of the paper is structured as follows. In Section 2 we introduce the model. In Section 3 we solve for equilibrium asset prices, as well as for the informed agent’s information acquisition choices. In Section 4 we present our main results, on the welfare implications of indexing. Section 6 concludes.

2 The model

We assume that there is a continuum of agents, indexed by the unit interval, $i \in [0, 1]$. All agents have CARA preferences with risk-aversion parameter γ .

There are two risky assets available for trading, with payoffs X_1 and X_2 , where we assume that both assets follow a Gaussian distribution, $X_j \sim \mathcal{N}(\mu_j, \sigma_j^2)$, and that they are uncorrelated. For simplicity in the presentation, we shall assume that $\mu_1 = \mu_2 = \mu_x$, and $\sigma_1 = \sigma_2 = \sigma_x$. We will use τ_x to denote the precision of random variable x , i.e. $\tau_x = 1/\sigma_x^2$. The price of asset j will be denoted by P_j , and $P = (P_1, P_2)$

Agents have endowments of the two risky assets. Each agent i has a non-random endowment of the risky asset j of \bar{s} , and a random endowment shock $e_{ij} = Z_j + u_{ij}$. The random endowment shock is privately observed by agent i . We assume that $Z_j \sim \mathcal{N}(0, \sigma_z^2)$, and that $u_{ij} \sim \mathcal{N}(0, \sigma_u^2)$. One can think about Z_j as aggregate endowment shocks, which

motivate trade, and u_{ij} as idiosyncratic shocks that prevent agents from observing the aggregate shock. We let θ_{ij} denote the trade on asset j by agent i . Furthermore, let θ_i , e_i and \bar{s}_i denote the vectors of trading strategies and endowment shocks for agent i .

There are three types of traders in our model. The first set are informed agents, who observe private signals prior to trading in financial markets. We think of institutional investors as playing this role, although we step away from a formal model of the mutual fund/hedge fund industry (see Garcia and Vanden, 2009). The rest of agents are uninformed, mimicking the retail sector. There are two types of uninformed traders: “active traders,” who are unconstrained, and “indexers,” who decide to only invest in the market portfolio. Given our symmetry assumptions, the indexers will invest in equal amounts of assets 1 and 2. At this point we do not provide a micro-foundation for the existence of the different groups of investors, but rather just assume there is a mass λ_I of informed agents, and, similarly, we let λ_U denote the number of active uninformed agents. We will denote the number of indexers by η , which will be our key parameter, since our focus will be on the impact of increasing indexing, η , on asset prices and on welfare. We note that $\lambda_I + \lambda_U + \eta = 1$.

Active agents can acquire information about the payoffs of the assets prior to trading. In particular, agent i can acquire information about the “market portfolio” $X_m \equiv \frac{1}{\sqrt{2}}(X_1 + X_2)$, or about the “relative value” of the two assets, $X_s \equiv \frac{1}{\sqrt{2}}(X_1 - X_2)$, which we will refer to as the “spread portfolio.” We define the aggregate supply of the market and spread assets as expected: $\bar{s}_m = \sqrt{2}\bar{s}$, and $\bar{s}_s = 0$. In particular, agent i will observe the two signals

$$\begin{aligned} Y_{im} &= \frac{1}{\sqrt{2}}(X_1 + X_2) + \epsilon_{im}, \\ Y_{is} &= \frac{1}{\sqrt{2}}(X_1 - X_2) + \epsilon_{is}, \end{aligned}$$

where $\epsilon_{ij} \sim \mathcal{N}(0, \tau_{ij}^{-1})$ for $j = m, s$. We will use \mathcal{F}_i to denote the information set agent i has at the time of trading, which will consist of private signals Y_{im} and Y_{is} , if informed, as well

as the prices of both the market and spread assets. Agents must pay a cost $\kappa(\tau_{im}, \tau_{is})$ to observe these signals. Let κ_j denote the first derivative of κ with respect to τ_{ij} , for $j = m, s$, and similarly for higher order derivatives. We will make the following assumptions on the costs of acquiring information.

Assumption 1. *The information costs function satisfies:*

1. $\kappa_m \geq 0$, $\kappa_s \geq 0$, $\kappa_{mm} > 0$, $\kappa_{ss} > 0$, $\kappa_{ms} > 0$, and $\kappa_{mm}\kappa_{ss} - \kappa_{ms}^2 \geq 0$ where the inequalities are strict except for at $(\tau_{im}, \tau_{is}) = (0, 0)$.
2. If $\tilde{\tau}_{im} \geq \tau_{im}$ and $\tilde{\tau}_{is} \leq \tau_{is}$, then $\kappa_m(\tilde{\tau}_{im}, \tilde{\tau}_{is}) \leq \kappa_m(\tau_{im}, \tau_{is})$ implies $\kappa_s(\tilde{\tau}_{im}, \tilde{\tau}_{is}) \leq \kappa_s(\tau_{im}, \tau_{is})$ and $\kappa_s(\tilde{\tau}_{im}, \tilde{\tau}_{is}) \geq \kappa_s(\tau_{im}, \tau_{is})$ implies $\kappa_m(\tilde{\tau}_{im}, \tilde{\tau}_{is}) \geq \kappa_m(\tau_{im}, \tau_{is})$, along with the symmetric property.

The first set of assumptions are quite mild. The second assumption makes the marginal cost of information about the market portfolio more responsive to changes in the amount of information collected about the market portfolio (τ_{ϵ_m}) than the marginal cost of information about the spread asset.⁴

As usual in this class of models, we will look for linear equilibria, where prices have the form

$$P = A + BX - DZ; \tag{1}$$

for some 2×2 matrices B and D , and 2×1 vector A .

The following definition is standard.

Definition 1. *A rational expectations equilibrium is a set of trading strategies $\{\theta_{im}, \theta_{is}\}_{i \in [0,1]}$, a price function $P(X, Z)$, and a set of signal precisions chosen by the informed agents, $\{\tau_{im}, \tau_{is}\}_{i \in [0,1]}$, such that*

⁴A simple example of a cost function satisfying these properties is $\kappa(\tau_{im}, \tau_{is}) = \frac{1}{2}\tau_{im}^2 + \frac{1}{2}\tau_{is}^2 + \delta\tau_{im}\tau_{is}$ for any $\delta \leq 1$.

1. *Markets clear:*

$$\int_0^1 \theta_{ik} di = \bar{s}_k; \quad (2)$$

for $k = m, s$.

2. *Taking the price function in (1) as given, agent i 's trading strategy is optimal:*

$$(\theta_{im}, \theta_{is}) \in \arg \max_{(\hat{\theta}_{im}, \hat{\theta}_{is})} \mathbb{E} [u(W_i) | \mathcal{F}_i]. \quad (3)$$

3. *Agents choices of information are optimal:*

$$(\tau_{im}, \tau_{is}) \in \arg \max_{(\theta_{im}, \theta_{is})} \mathbb{E} [u(W_i)]. \quad (4)$$

The equilibrium definition is standard, and follows the competitive rational expectations models started with Grossman and Stiglitz (1980). The assumption of a continuum of agents, and the multi-asset structure, makes our model very close to Admati (1985), which the added stage of information acquisition as in Verrecchia (1982), and the source of “noise” being driven by privately observed endowment shocks, following Diamond and Verrecchia (1981), Ganguli and Yang (2009) and Manzano and Vives (2011).

There are a few features worth remarking upon. First is the fact that the model’s “noise,” which prevents prices from being fully revealing (Black, 1986) comes from endowment shocks to agents’ private portfolios. This means that all agents have some private signal, their individual endowment shock e_i that they receive, which is correlated with the aggregate shock Z . More importantly for our purposes, explicitly modelling the source of noise allows us to look at welfare issues, since we can explicitly evaluate the expected utility of all players. The alternative of using “noise traders,” as in Kyle (1985), introduces the decision of whether to include them in welfare calculations, and/or how much weight to give them (Leland, 1992).

It is also worth noticing that while we have introduced the model as a two-asset economy, the symmetry and independent assumptions we make essentially make the problem decouple into two one-asset economies at the trading stage. The two assets will be related via the information acquisition choices, by design of the signal structure available.

3 Equilibrium asset prices

In Section 3.1 we study the equilibrium in our model at the trading stage, fixing the number of informed traders. In Section 3.2 we solve for the information acquisition decision of the informed active traders. Proofs are included in the appendix.

3.1 Equilibrium at the trading stage

For presentation purposes, it is useful to express all portfolio positions in terms of the spread and market assets, paying X_m and X_s respectively. Clearly the span faced by agents, as well as the informational content of prices, is unchanged. Note that $\text{var}(X_m) = \text{var}(X_s) = \sigma_x^2$, $\text{cov}(X_m, X_s) = 0$, $\mathbb{E}[X_m] = \sqrt{2}\mu_x$ and $\mathbb{E}[X_s] = 0$. Each agent has an endowment $\bar{s}_m = \sqrt{2}\bar{s}$ of the market asset, and no endowment of the spread asset.

Writing θ_{im} and θ_{is} for agent i 's positions in the market and spread asset, and defining $e_{im} \equiv \frac{1}{\sqrt{2}}(e_{i1} + e_{2i})$ and $e_{is} \equiv \frac{1}{\sqrt{2}}(e_{i1} - e_{2i})$, we can write agent i 's terminal wealth as

$$W_i = \left(\sqrt{2}\bar{s} + e_{im}\right) P_m + e_{is} P_s + (\theta_{im} + e_{im})(X_m - P_m) + (\theta_{is} + e_{is})(X_s - P_s).$$

The above expression shows an agent's terminal wealth being driven by the trading profits in the market and spread asset (the last two terms), as well as the endowment shocks on the market, and the spread asset (the first two terms). Note that under our normalization(s), we have $\text{var}(e_{im}) = \text{var}(e_{is}) = \sigma_z^2 + \sigma_u^2$, $\text{cov}(e_{im}, e_{is}) = 0$, $\mathbb{E}[e_{im}] = \mathbb{E}[e_{is}] = 0$.

Likewise, define the aggregate market/spread endowment shocks as $Z_m = \frac{1}{\sqrt{2}}(Z_1 + Z_2)$ and $Z_s = \frac{1}{\sqrt{2}}(Z_1 - Z_2)$, and $u_{im} = \frac{1}{\sqrt{2}}(u_{i1} + u_{i2})$ and $u_{is} = \frac{1}{\sqrt{2}}(u_{i1} - u_{i2})$. Note that $e_{ij} = Z_j + u_{ij}$ for $j = m, s$. These definitions allow us to write the price of the market and spread assets as simple functions of these aggregate quantities. In particular, we conjecture that the prices of the market portfolio and the spread asset are given by

$$P_m = \mu_m + bX_m - dZ_m,$$

$$P_s = aX_s - cZ_s.$$

The next Proposition solves for the unique stable equilibrium in our model.

Proposition 1. *Assume that $4\gamma^2(\tau_z^{-1} + \tau_u^{-1}) < \tau_x$. Further assume that active traders have signals with precisions $\tau_{im} = \tau_{em}$ and $\tau_{is} = \tau_{es}$ for all $i \in [0, \lambda_I]$. Then a symmetric equilibrium at the trading stage exists, where the price coefficients satisfy:*

$$\rho_m \equiv \frac{b}{d} = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u}\right)^2 - \lambda_I \frac{\tau_{em}}{\tau_u}} \quad (5)$$

$$\rho_s \equiv \frac{a}{c} = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u}\right)^2 - \frac{\lambda_I}{(1-\eta)} \frac{\tau_{es}}{\tau_u}} \quad (6)$$

$$b = \frac{\rho_m^2(\tau_z + \tau_u) + \lambda_I \tau_{em}}{\tau_x + \rho_m^2(\tau_z + \tau_u) + \lambda_I \tau_{em}} \quad (7)$$

$$a = \frac{\rho_s^2(\tau_z + \tau_u) + \frac{\lambda_I}{1-\eta} \tau_{es}}{\tau_x + \rho_s^2(\tau_z + \tau_u) + \frac{\lambda_I}{1-\eta} \tau_{es}} \quad (8)$$

$$\mu_m = \frac{\sqrt{2}(\mu_x \tau_x - \gamma \bar{s})}{\tau_x + \rho_m^2(\tau_z + \tau_u) + \lambda_I \tau_{em}} \quad (9)$$

Equilibrium prices satisfy

$$\mathbb{E}[X_m - P_m] = \sqrt{2}\gamma \bar{s} \text{cov}(X_m - P_m, X_m) \quad (10)$$

The equilibrium in Proposition 1 generalizes that in Ganguli and Yang (2009) and Manzano and Vives (2011) to the case of two assets. As in their work, it is worth noticing that there are multiple equilibria at the trading stage. The equilibrium in Proposition 1 is the unique linear equilibria that is stable, see the discussion in Manzano and Vives (2011). While our model does require the parametric assumption $4\gamma^2(\tau_z^{-1} + \tau_u^{-1}) < \tau_x$ for existence, Manzano and Vives (2011), in a closely related model, show that non-existence is not generally an issue.⁵ The expressions in the Proposition highlight how the model “decouples” into two separate “one-asset” economies, one for the market asset, and one for the spread asset.

Two key equilibrium parameters are the relative price coefficients $\rho_m = b/d$ and $\rho_s = a/c$. We note that ρ_m measures the price informativeness of the market portfolio, namely

$$\text{var}(X_m|\mathcal{F}_u)^{-1} = \tau_x + \rho_m^2(\tau_z + \tau_u) \quad (11)$$

whereas ρ_s measures price informativeness in the spread asset market,

$$\text{var}(X_s|\mathcal{F}_u)^{-1} = \tau_x + \rho_s^2(\tau_z + \tau_u) \quad (12)$$

where $\mathcal{F}_u = \sigma(P_m, P_s, e_{im}, e_{is})$ is the information set of an uninformed agents.

Indexing only directly affects the spread asset in our model. This is rather intuitive, as the problem decouples, and indexing does not affect how agents trade in the market portfolio. On the other hand, indexing does take away traders from the spread asset, by assumption. The main effect is to take away “noise” from this spread asset, as the endowment shocks of the indexers will not move trading in the spread asset. As we see from (6), indexing will increase ρ_s , and as a consequence price revelation in the spread asset.

⁵The main extension in Manzano and Vives (2011) on Ganguli and Yang (2009) is that they allow for the error terms in the trader’s signals to be correlated. Non-zero correlation eliminates the existence issues in our model. Since our focus is on welfare, we choose to study the slightly more tractable model with conditionally independent estimation errors.

We remark on the drivers of the equity premium in (10): aggregate risk-aversion and total supply scale up the “market risk” of the index portfolio, the covariance of trading profits $X_m - P_m$ with final payoffs X_m . Using the equilibrium expressions, one can easily verify that

$$\mathbb{E}[X_m - P_m] = \frac{\sqrt{2}\gamma\bar{s}\tau_x}{\tau_x + \rho_m^2(\tau_z + \tau_u) + \lambda_I\tau_{em}}. \quad (13)$$

The risk premia in our model is driven by the amount of aggregate risk, as measured by \bar{s} and τ_x , as well as the risk tolerance in the economy, γ . Informed trading affects the risk premia by changing the amount of information revealed by prices. The term in the denominator of (13) is the conditional precision of the market asset, as in (11), plus a term that increases with the size of the informed population as well as with the quality of their signals. Rather intuitively, the more information revelation, the less risk agents face when investing in the market portfolio, which in turn lowers the equilibrium risk premia. It is important to note how the only way that indexing will affect the market portfolio is via the information acquisition decision.

3.2 Information allocations

In this section we study the information acquisition decisions of the informed active traders. At the heart of our paper is to understand how “smart” money reacts to fact that more investors are indexing. The next proposition characterizes the information acquisition decision of informed agents in our model.

Proposition 2. *If information production costs are sufficiently high, then a symmetric equilibrium at the information acquisition stage exists, and it is unique. The active informed*

agents information acquisition decisions are $\tau_{im} = \tau_{em}$ and $\tau_{is} = \tau_{es}$, where (τ_{em}, τ_{es}) solve:

$$2\gamma\kappa_m(\tau_{em}, \tau_{es}) = \frac{1}{\tau_x + \rho_m^2(\tau_z + \tau_u) + \tau_{em}} \quad (14)$$

$$2\gamma\kappa_s(\tau_{em}, \tau_{es}) = \frac{1}{\tau_x + \rho_s^2(\tau_z + \tau_u) + \tau_{es}}. \quad (15)$$

Proposition 2 closes the model at the information gathering stage. Equations (14) and (15) are simply the first-order conditions to the informed agent's decision regarding the precision of their signals. Agents' choices are driven by the trade-off between the extra precision's effect on ex-ante utility, the right-hand side terms in (14) and (15), and the marginal cost of acquiring such precision. We remark that the presence of the second term in the equilibrium conditions for ρ_m and ρ_s , see equations (5) and (6), precludes closed-form solutions.⁶

Lemma 2 in the proof shows how the spread and market asset prices interact in the information acquisition decision. The information gathered on the market (spread) asset decreases as the market (spread) asset becomes more informative. This is the standard substitution effect from Grossman and Stiglitz (1980): more information revelation makes becoming informed less valuable. On the other hand, the precision of the market (spread) signal increases as the spread (market) becomes more informative. Our model thus generates complementarities among the spread and market asset with respect to the information acquisition of agents: more information revelation in one asset makes agents more willing to engage in information acquisition activities in the other asset. This will be the key channel via which indexing will affect the market asset.

⁶In the quadratic cost function case, $\kappa(\tau_{im}, \tau_{is}) = \frac{1}{2}\tau_{im}^2 + \frac{1}{2}\tau_{is}^2 + \delta\tau_{im}\tau_{is}$, one can verify (14) and (15) reduce to polynomials of degree four.

4 Indexing, asset prices and welfare

The focus of our paper is on the role of indexing on equilibrium asset prices. We start by analyzing what happens to the information acquired by agents as indexing increases.

Proposition 3. *Consider two different levels of indexing η^* and η' , with associated equilibrium price coefficients (ρ_m^*, ρ_s^*) and (ρ_m', ρ_s') . If $\eta^* > \eta'$, then $\rho_m^* \geq \rho_m'$, and $\rho_s^* \geq \rho_s'$, with strict inequality if $\rho_m^* > 0$.*

Proposition 3 establishes that information revelation increases in *both* the market and spread assets as indexing increases. Intuitively, when agents decide to index, the “noise” in the spread asset, created by the endowment shocks, is reduced. This makes prices more informative, which, in turn, decreases the information production by institutional investors in the spread asset, and due to the complementarities discussed above, it increases their investment in the market signals. As a consequence prices in the market asset become more informative.

Having established that more indexing makes prices more informative, we turn next to the welfare consequences of indexing. Among the contributions of our paper is to produce tractable results on the value of information. Vives and Medrano (2004) argue that “the expressions for the expected utility of a hedger ... are complicated,” whereas Kurlat and Veldkamp (2015) write that “there is no closed-form expression for investor welfare.” The complications, common to our model as well, stem from the role of the endowment shocks as signals regarding price movements, on top of the standard risk sharing role that motivates trade.

Intuitively, increasing indexing moves more informed trading into the market portfolio, which hurts the indexers, who are at an informational disadvantage. On the other hand, the increase in informed trading may decrease the dependence of the price P_m on the aggregate endowment shock Z_m , represented by the coefficient d . Economically, as price informative-

ness ρ_m increases, investor demand becomes more responsive to the price, and so the price can in turn become less responsive to the endowment shock Z_m while still satisfying market clearing. When the increase in informed trading reduces the price's dependence on Z_m , this is good news for all agents, including indexers, since they bear less risk when trading (Hirshleifer (1971)).

The following result signs the net effect of these two offsetting forces, and establishes that an increase in informed trading indeed reduces indexer welfare. As the proof demonstrates, the importance of the second effect above is limited by the fact that all agents have the same average endowment (\bar{s}) of the risky assets 1, 2, which in turn limits the amount of endowment-based trading that indexers need to carry out.

Proposition 4. *The expected utility of an indexer decreases in the size of the indexing population η .*

We conclude by providing a numerical example. The example illustrates our main empirical predictions, and it will allow us to assess whether the result in Proposition 4 carries over to other market participants, i.e. the informed investors and the active uninformed agents.⁷

We consider the following parameter values, which ensure existence of equilibria, but are otherwise arbitrary:⁸ $\tau_x = 1$, $\tau_z = 3$, $\tau_u = 5$, $\bar{s} = 1$, $\mu_x = 1$, $\gamma = 0.2$, $\kappa = 0.01$, $\lambda_I = 0.1$. Figure 1 plots the price informativeness for the market and spread asset as a function of the mass of indexers. As established in Proposition 3, more indexing results in more informative prices, and the effect is more pronounced for the spread asset, as the removal of the noise generated by the endowment shocks makes the informed trading more salient.

Figure 2 plots the equilibrium market risk premia as a function of the mass of indexers, which monotonically decreases as more indexers enter the market. From the closed-form

⁷The expected utilities for both the informed and active uninformed are analytically significantly more challenging than that of the indexers.

⁸We have considered other parameter constellations with similar qualitative results.

expression in (10), we can see that the channel via which indexing affects the market risk-premia is the information gathering activities of the informed agents. Since noise dries out in the spread asset as more uninformed agents leave the spread asset market, the smart money focuses on the market asset, increasing their information acquisition, which in turn makes prices more informative, and, as a consequence, they lower the conditional variance of the market, and its equilibrium risk premia.

While Proposition 4 clearly argues for a negative externality coming from indexing on other indexers, it leaves open the possibility that informed and active uninformed agents could benefit from more indexing. We investigate this question numerically, and plot in Figure 3 the expected utilities of indexers (solid line), uninformed active traders (dotted line), and informed agents (dashed line). The negative externality from Proposition 4 of more indexing on indexers seems to be more general, in that all agents are made worse off in our numerical analysis.

We next investigate the effect of indexing on the correlation of trading profits from the informed agents with the market. The rise in index investing over the last 2-3 decades has been accompanied by an increase in the correlation of mutual fund returns with the market (Stambaugh, 2014). In Figure 4 we plot the relationship between indexing and the correlation between the trading profits of informed agents and the market. In our model more indexing makes the informed intensify their information gathering activities in the market, which in turn makes their trading profits more correlated with market returns.

5 Discussion

In addition to showing that, in equilibrium, indexing reduces the welfare of retail investors, our analysis produces a number of empirical implications. First, indexing reduces the market risk premium. While empirically the evidence supports a correlation between the amount of

indexing and the equity risk premium, it is hard to argue causality using empirical methods. Our model explicitly links the increase in passive investment to a lower equity risk premium.

Indexing leads to higher levels of “price efficiency,” as measured by the ability of prices to predict future cash flows. More indexing also leads to a shift in trading strategies of informed investors, who shift from trades based on cross-sectional “mispricing” to ones that focus instead on aggregate factors. These two implications are broadly consistent with the recent empirical literature on hedge funds and ETFs (Fung, Hsieh, Naik, and Ramadorai, 2008; Sun, Wang, and Zheng, 2012; Ramadorai, 2013; Pedersen, 2015; Israeli, Lee, and Sridharan, 2015).

Furthermore, the correlation between hedge fund returns with equity markets is significantly higher now than it was in the last two decades.⁹ This mirrors the evidence from the mutual fund literature (Stambaugh, 2014), and it is consistent with the numerical results presented in Figure 4.

6 Conclusion

Our paper studies a canonical model of trading among heterogeneously informed agents, extending the Admati (1985) framework to allow for endowment shocks. Our contribution is to show how indexing, committing to only trading on the market portfolio, affects asset prices and information allocations. The paper argues that indexing makes the smart money move away from relative performance trades (in the spread asset) and focus on trading the market portfolio. This makes prices more informative, and creates a negative externality on other uninformed traders.

⁹See “Hedge fund correlation risk alarms investors,” Financial Times, June 29th, 2014. AQR estimates the correlation to be above 0.9, up from about 0.6 in the 1990s.

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Appendix

Proof of Proposition 1. Optimal trading strategies for active traders are, for assets $j = m, s$,

$$\theta_{ij} = \frac{1}{\gamma} \frac{\mathbb{E}[X_j - P_j | \mathcal{F}_i]}{\text{var}(X_j | \mathcal{F}_i)} - e_{ij}, \quad (16)$$

where \mathcal{F}_i denotes the information set agent i has at the time of trading, which in particular includes the prices P_m, P_s .

Using the projection theorem, we first note that

$$\mathbb{E}[X_m | \mathcal{F}_i] = \frac{\tau_x \sqrt{2} \mu_x + \tau_z \left(\frac{b}{d}\right)^2 \frac{P_m - \mu_m}{b} + \tau_u \left(\frac{b}{d}\right)^2 \left(\frac{P_m - \mu_m}{b} + \frac{d}{b} e_{im}\right) + \tau_{im} Y_{im}}{\tau_x + \left(\frac{b}{d}\right)^2 (\tau_z + \tau_u) + \tau_{im}} \quad (17)$$

and

$$\mathbb{E}[X_s | \mathcal{F}_i] = \frac{\tau_z \left(\frac{a}{c}\right)^2 \frac{P_s}{a} + \tau_u \left(\frac{a}{c}\right)^2 \left(\frac{P_s}{a} + \frac{c}{a} e_{is}\right) + \tau_{is} Y_{is}}{\tau_x + \left(\frac{a}{c}\right)^2 (\tau_z + \tau_u) + \tau_{is}} \quad (18)$$

where $\mathcal{F}_i = \sigma(Y_{im}, Y_{is}, P_m, P_s, e_{im}, e_{is})$ is the information set of the informed agents.

We also note that

$$\text{var}(X_m | \mathcal{F}_i)^{-1} = \tau_x + \rho_m^2 (\tau_z + \tau_u) + \tau_{im} \quad (19)$$

$$\text{var}(X_s | \mathcal{F}_i)^{-1} = \tau_x + \rho_s^2 (\tau_z + \tau_u) + \tau_{is} \quad (20)$$

The expressions for the uninformed agents follow setting $\tau_{im} = \tau_{is} = 0$ in (17)-(20).

Using (16), (17) and (19), one can write the informed agents' trading strategy in the market asset as

$$\theta_{im} = \frac{1}{\gamma} \left(K_1 + \rho_m^2 (\tau_z + \tau_u) \frac{P_m}{b} + \tau_u \rho_m e_{im} + \tau_{im} Y_{im} - (\tau_x + \rho_m^2 (\tau_z + \tau_u) + \tau_{im}) P_m \right) - e_{im},$$

where $K_1 = \tau_x \sqrt{2} \mu_x \mu_x - \mu_m \rho_m^2 (\tau_z + \tau_u) / b$.

Similarly, the trading strategy for the uninformed active agents, as well as the indexers, is given by

$$\theta_{im} = \frac{1}{\gamma} \left(K_1 + \rho_m^2 (\tau_z + \tau_u) \frac{P_m}{b} + \tau_u \rho_m e_{im} - (\tau_x + \rho_m^2 (\tau_z + \tau_u)) P_m \right) - e_{im}.$$

Integrating the demand terms, the market clearing condition for the market asset is

$$\lambda_I \tau_{im} X_m + K_1 + \rho_m^2 (\tau_z + \tau_u) \left(\frac{1}{b} - 1 \right) P_m - (\tau_x + \lambda_I \tau_{im}) P_m + (\tau_u \rho_m - \gamma) Z_m = \sqrt{2} \gamma \bar{s}.$$

“Matching coefficients” we have three equilibrium conditions:

$$\begin{aligned} b \left(\rho_m^2 (\tau_z + \tau_u) \left(\frac{1}{b} - 1 \right) - (\tau_x + \lambda_I \tau_{im}) \right) + \lambda_I \tau_{im} &= 0 \\ -d \left(\rho_m^2 (\tau_z + \tau_u) \left(\frac{1}{b} - 1 \right) - (\tau_x + \lambda_I \tau_{im}) \right) + (\tau_u \rho_m - \gamma) &= 0 \\ K_1 + \mu_m \left(\rho_m^2 (\tau_z + \tau_u) \left(\frac{1}{b} - 1 \right) - (\tau_x + \lambda_I \tau_{im}) \right) - \sqrt{2} \gamma \bar{s} &= 0 \end{aligned}$$

Combining the first two of these equations pins down the information revelation parameter $\rho_m = b/d$ as the solution to

$$\tau_u \rho_m^2 - \gamma \rho_m + \lambda_I \tau_{im} = 0. \quad (21)$$

Expression (7) follows easily from the first equation. In order to prove (10), we first note that, taking expectations from the first-order conditions of an agent’s optimal portfolio choice (16), aggregating, and imposing market-clearing, we have

$$\frac{1}{\gamma} \mathbb{E}[X_m - P_m] \left((\eta + \lambda_U) \text{var}(X_m | \mathcal{F}_u)^{-1} + \lambda_I \text{var}(X_m | \mathcal{F}_i)^{-1} \right) = \sqrt{2} \bar{s}$$

Next, using (7) and (19), we have

$$(\eta + \lambda_U) \text{var}(X_m | \mathcal{F}_u)^{-1} + \lambda_I \text{var}(X_m | \mathcal{F}_i)^{-1} = \frac{\tau_x}{1 - b},$$

and so

$$\mathbb{E}[X_m - P_m] = \sqrt{2} \gamma \bar{s} \frac{1 - b}{\tau_x}$$

Since $\text{cov}(X_m - P_m, X_m) = \frac{1-b}{\tau_x}$, this establishes (10). Moreover, this same equation gives

$$\mu_m = (1 - b) \left(\sqrt{2} \mu_x - \frac{\sqrt{2} \gamma \bar{s}}{\tau_x} \right),$$

which combines with (7) to deliver (9).

We now turn to the spread asset. Using (16), (18) and (20) one can write the informed

agents' trading strategy in the market asset as

$$\theta_{is} = \frac{1}{\gamma} \left(\rho_s^2(\tau_z + \tau_u) \frac{P_s}{a} + \tau_u \rho_m e_{is} + \tau_{is} Y_{is} - (\tau_x + \rho_s^2(\tau_z + \tau_u) + \tau_{is}) P_s \right) - e_{is}.$$

Similarly, the trading strategy for the uninformed active agents is given by

$$\theta_{is} = \frac{1}{\gamma} \left(\rho_s^2(\tau_z + \tau_u) \frac{P_s}{a} + \tau_u \rho_s e_{is} - (\tau_x + \rho_s^2(\tau_z + \tau_u)) P_s \right) - e_{is}.$$

Integrating the demand terms, the market clearing condition for the spread asset is

$$\lambda_I \tau_{is} X_s + (1 - \eta) \rho_s^2(\tau_z + \tau_u) \left(\frac{1}{a} - 1 \right) P_s - ((1 - \eta) \tau_x + \lambda_I \tau_{is}) P_s + (1 - \eta)(\tau_u \rho_s - \gamma) Z_s = 0.$$

“Matching coefficients” we have two equilibrium conditions:

$$\begin{aligned} a \left(\rho_s^2(\tau_z + \tau_u) \left(\frac{1}{a} - 1 \right) - \left(\tau_x + \frac{\lambda_I}{1 - \eta} \tau_{is} \right) \right) + \frac{\lambda_I}{1 - \eta} \tau_{is} &= 0 \\ -c \left(\rho_s^2(\tau_z + \tau_u) \left(\frac{1}{a} - 1 \right) - \left(\tau_x + \frac{\lambda_I}{1 - \eta} \tau_{is} \right) \right) + (\tau_u \rho_s - \gamma) &= 0. \end{aligned}$$

Hence the information revelation parameter $\rho_s = a/c$ as the solution to

$$\tau_u \rho_s^2 - \gamma \rho_s + \frac{\lambda_I}{1 - \eta} \tau_{is} = 0. \quad (22)$$

Expression (8) follows easily from the first equation, completing the proof. ■

Proof of Proposition 2. The final wealth of agent i , given the optimal trading strategies (16) is given by

$$W_i = \left(\sqrt{2\bar{s}} + e_{im} \right) P_m + e_{is} P_s + \frac{\mathbb{E}[X_m - P_m | \mathcal{F}_i] (X_m - P_m)}{\gamma \text{var}(X_j | \mathcal{F}_i)} + \frac{\mathbb{E}[X_s - P_s | \mathcal{F}_i] (X_s - P_s)}{\gamma \text{var}(X_j | \mathcal{F}_i)}.$$

So the expected utility of an active trader, given his information set at the time of trading, is given by

$$\mathbb{E}[-\exp(-\gamma W_i) | \mathcal{F}_i] = -\exp \left(-\gamma \left(\sqrt{2}(\bar{s} + e_{im}) P_m + e_{is} P_s + \frac{1}{2} \frac{\mathbb{E}[X_m - P_m | \mathcal{F}_i]^2}{\gamma \text{var}(X_j | \mathcal{F}_i)} + \frac{1}{2} \frac{\mathbb{E}[X_s - P_s | \mathcal{F}_i]^2}{\gamma \text{var}(X_j | \mathcal{F}_i)} \right) \right). \quad (23)$$

From Proposition 1 we know that $X_m - P_m$ and $X_s - P_s$ are independent, so in order to

evaluate (23) we first compute

$$\mathbb{E} \left[\exp \left(-\frac{1}{2} \frac{\mathbb{E}[X_j - P_j | \mathcal{F}_i]^2}{\text{var}(X_j | \mathcal{F}_i)} \right) \middle| \mathcal{F}_u \right], \quad (24)$$

for $j = m, s$, where $\mathcal{F}_u = \sigma(P_m, P_s, e_m, e_s)$ is the information set of the uninformed agents, and \mathcal{F}_i is either \mathcal{F}_u or the filtration observed by the informed $\mathcal{F}_i = \sigma(Y_m, Y_s, P_m, P_s, e_m, e_s)$.

In order to evaluate (24) we use the standard linear-quadratic formula¹⁰ letting $\xi_i = \mathbb{E}[X_j - P_j | \mathcal{F}_i]$ $A = -1/(2\text{var}(X_j | \mathcal{F}_i))$, and so the expectation in (24) equals:

$$\mathbb{E} \left[\exp(\xi_i^2 A) \right] = |1 - 2\text{var}(\xi_i | \mathcal{F}_u)A|^{-1/2} \exp(\mu_j^2 A + 2\mu_j^2 A^2 \text{var}(\xi_i | \mathcal{F}_u)(1 - 2\text{var}(\xi_i | \mathcal{F}_u)A)^{-1}). \quad (26)$$

Using the law of the total variance we have that

$$\text{var}(X_j - P_j | \mathcal{F}_u) = \text{var}(\mathbb{E}[X_j - P_j | \mathcal{F}_i] | \mathcal{F}_u) + \mathbb{E}[\text{var}(X_j - P_j | \mathcal{F}_i) | \mathcal{F}_u]$$

which implies

$$\text{var}(\xi_i | \mathcal{F}_u) = \text{var}(X_j | \mathcal{F}_u) - \text{var}(X_j | \mathcal{F}_i) \quad (27)$$

$$1 - 2\text{var}(\xi_i | \mathcal{F}_u)A = \frac{\text{var}(X_j | \mathcal{F}_u)}{\text{var}(X_j | \mathcal{F}_i)}. \quad (28)$$

Substituting into (26) gives

$$\mathbb{E} \left[\exp \left(-\frac{1}{2} \frac{\mathbb{E}[X_j - P_j | \mathcal{F}_i]^2}{\gamma \text{var}(X_j | \mathcal{F}_i)} \right) \middle| \mathcal{F}_u \right] = \sqrt{\frac{\text{var}(X_j | \mathcal{F}_i)}{\text{var}(X_j | \mathcal{F}_u)}} \exp \left(-\frac{1}{2} \frac{\mathbb{E}[X_j - P_j | \mathcal{F}_u]^2}{\text{var}(X_j | \mathcal{F}_u)} \right). \quad (29)$$

Noting that the the exponential term in (29) is independent of the information acquisition

¹⁰The following lemma is a standard result on multivariate normal random variables.

Lemma 1. *Let $X \in \mathbb{R}^n$ be a normally distributed random vector with mean μ and variance-covariance matrix Σ . Let $b \in \mathbb{R}^n$ be a given vector, and $A \in \mathbb{R}^{n \times n}$ a symmetric matrix. If $I - 2\Sigma A$ is positive definite, then $\mathbb{E}[\exp(b^\top X + X^\top A X)]$ is well defined, and given by:*

$$\mathbb{E}[\exp((b^\top X + X^\top A X))] = |I - 2\Sigma A|^{-1/2} \exp \left(b^\top \mu + \mu^\top A \mu + \frac{1}{2}(b + 2A\mu)^\top (I - 2\Sigma A)^{-1} \Sigma (b + 2A\mu) \right) \quad (25)$$

decision, it is straightforward to check that the informed agents are maximizing

$$- \exp(\gamma \kappa(\tau_{im}, \tau_{is})) \sqrt{\frac{\tau_x + \rho_m^2(\tau_z + \tau_u)}{\tau_x + \rho_m^2(\tau_z + \tau_u) + \tau_{im}}} \sqrt{\frac{\tau_x + \rho_s^2(\tau_z + \tau_u)}{\tau_x + \rho_s^2(\tau_z + \tau_u) + \tau_{is}}}, \quad (30)$$

where we have used (11)-(12) and (19)-(20) in (29).

The first-order conditions to (30) are (14) and (15). We note that the Hessian is positive definite, so there is a unique pair (τ_{im}, τ_{is}) that solves (30).¹¹ Let this unique pair be denoted by $(\tau_{im}, \tau_{is}) = (g_m(\rho_m, \rho_s), g_s(\rho_m, \rho_s))$.

Lemma 2. *The functions $g_m(\rho_m, \rho_s)$ and $g_s(\rho_m, \rho_s)$ satisfy:*

$$\frac{\partial g_m}{\partial \rho_m} < 0, \quad \frac{\partial g_m}{\partial \rho_s} > 0, \quad \frac{\partial g_s}{\partial \rho_s} < 0, \quad \frac{\partial g_s}{\partial \rho_m} > 0.$$

Proof of Lemma 2. First, we establish $\frac{\partial g_m}{\partial \rho_m} < 0$. Suppose to the contrary that $\frac{\partial g_m}{\partial \rho_m} \geq 0$. Hence there exist ρ_s, ρ_m and $\tilde{\rho}_m > \rho_m$ such that $g_m(\tilde{\rho}_m, \rho_s) \geq g_m(\rho_m, \rho_s)$. From the first-order condition (14), it follows that $\kappa_m(g_m(\tilde{\rho}_m, \rho_s), g_s(\tilde{\rho}_m, \rho_s)) < \kappa_m(g_m(\rho_m, \rho_s), g_s(\rho_m, \rho_s))$. Hence $g_s(\tilde{\rho}_m, \rho_s) < g_s(\rho_m, \rho_s)$. On the one hand, Assumption 1 implies $\kappa_s(g_m(\tilde{\rho}_m, \rho_s), g_s(\tilde{\rho}_m, \rho_s)) \leq \kappa_s(g_m(\rho_m, \rho_s), g_s(\rho_m, \rho_s))$. On the other hand, (15) implies $\kappa_s(g_m(\tilde{\rho}_m, \rho_s), g_s(\tilde{\rho}_m, \rho_s)) > \kappa_s(g_m(\rho_m, \rho_s), g_s(\rho_m, \rho_s))$. The contradiction establishes the result.

Second, we establish $\frac{\partial g_m}{\partial \rho_s} > 0$. Suppose to the contrary that $\frac{\partial g_m}{\partial \rho_s} \leq 0$. Hence there exist ρ_m, ρ_s and $\tilde{\rho}_s > \rho_s$ such that $g_m(\rho_m, \tilde{\rho}_s) \leq g_m(\rho_m, \rho_s)$. From (14), we know $\kappa_m(g_m(\rho_m, \tilde{\rho}_s), g_s(\rho_m, \tilde{\rho}_s)) \geq \kappa_m(g_m(\rho_m, \rho_s), g_s(\rho_m, \rho_s))$. Hence $g_s(\rho_m, \tilde{\rho}_s) \geq g_s(\rho_m, \rho_s)$, and from Assumption 1, $\kappa_s(g_m(\rho_m, \tilde{\rho}_s), g_s(\rho_m, \tilde{\rho}_s)) \leq \kappa_s(g_m(\rho_m, \rho_s), g_s(\rho_m, \rho_s))$. From (15), it follows that $g_s(\rho_m, \tilde{\rho}_s) < g_s(\rho_m, \rho_s)$, a contradiction, establishing the result.

The remaining two results follow by analogous arguments. ■

In order to proof uniqueness and existence, first define

$$\bar{\tau}_{em} = \frac{\tau_u}{(1 - \eta)\lambda_I} \left(\frac{\gamma}{2\tau_u} \right)^2$$

¹¹The Hessian is given by

$$2 \begin{pmatrix} \gamma \kappa_{mm}(\tau_{im}, \tau_{is}) + \frac{\tau_x + \rho_m^2(\tau_z + \tau_u)}{(\tau_x + \rho_m^2(\tau_z + \tau_u) + \tau_{im})^3} & \gamma \kappa_{ms}(\tau_{im}, \tau_{is}) \\ \gamma \kappa_{ms}(\tau_{im}, \tau_{is}) & \gamma \kappa_{ss}(\tau_{im}, \tau_{is}) + \frac{\tau_x + \rho_s^2(\tau_z + \tau_u)}{(\tau_x + \rho_s^2(\tau_z + \tau_u) + \tau_{is})^3} \end{pmatrix},$$

which given Assumption 1 is indeed positive definite.

$$\bar{\tau}_{\epsilon s} = \frac{\tau_u}{\lambda_I} \left(\frac{\gamma}{2\tau_u} \right)^2$$

For any $\tau_{\epsilon m} \in [0, \bar{\tau}_{\epsilon m}]$, let $f_m(\tau_{\epsilon m})$ denote the price informativeness parameter in the market portfolio, i.e.

$$f_m(\tau_{\epsilon m}) = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u} \right)^2 - \lambda_I \frac{\tau_{\epsilon m}}{\tau_u}}.$$

The function f_m is strictly increasing in $\tau_{\epsilon m}$, and it satisfies $f_m(0) = 0$ and $f_m(\bar{\tau}_{\epsilon m}) = \frac{\gamma}{2\tau_u}$. Similarly, let $f_s(\tau_{\epsilon s}; \eta)$ denote the price informativeness parameter in the spread asset, i.e.

$$f_s(\tau_{\epsilon s}; \eta) = \frac{\gamma}{2\tau_u} - \sqrt{\left(\frac{\gamma}{2\tau_u} \right)^2 - \frac{\lambda_I}{(1-\eta)} \frac{\tau_{\epsilon s}}{\tau_u}}.$$

The function f_s is strictly increasing in $\tau_{\epsilon s}$, and it satisfies $f_s(0; \eta) = 0$ and $f_s(\bar{\tau}_{\epsilon s}; \eta) = \frac{\gamma}{2\tau_u}$ for all $\eta \in [0, 1)$. Furthermore, the function f_s is strictly increasing in η for all $\tau_{\epsilon s} \in (0, \bar{\tau}_{\epsilon s})$.

With the above notation, an equilibrium exists if there is a fixed point to the mapping

$$(f_m(g_m(\cdot, \cdot)), f_s(g_s(\cdot, \cdot))) : \left[0, \frac{\gamma}{2\tau_u} \right]^2 \rightarrow \left[0, \frac{\gamma}{2\tau_u} \right]^2 \quad (31)$$

For sufficiently high information production costs, it is clear that $g_m(\rho_m, \rho_s) < \bar{\tau}_{\epsilon m}$, $g_s(\rho_m, \rho_s) < \bar{\tau}_{\epsilon s}$, for all $(\rho_m, \rho_s) \in \left[0, \frac{\gamma}{2\tau_u} \right]^2$. Existence follows from Brouwer's fixed point theorem.

In order to proof uniqueness, we proceed by contradiction. Suppose that (ρ_m^*, ρ_s^*) and (ρ'_m, ρ'_s) both satisfy 31. From Lemma 2 we have that both $\rho_m^* \neq \rho'_m$ and $\rho_s^* \neq \rho'_s$. Assume, without loss of generality, that $\rho'_m > \rho_m^*$, so that $g_m(\rho'_m, \rho'_s) > g_m(\rho_m^*, \rho_s^*)$. From (14) we have that $\kappa_m(g_m(\rho'_m, \rho'_s), g_s(\rho'_m, \rho'_s)) < \kappa_m(g_m(\rho_m^*, \rho_s^*), g_s(\rho_m^*, \rho_s^*))$. This implies that $g_s(\rho'_m, \rho'_s) < g_s(\rho_m^*, \rho_s^*)$, so that $\rho'_s < \rho_s^*$. From (15) we conclude that $\kappa_s(g_m(\rho'_m, \rho'_s), g_s(\rho'_m, \rho'_s)) > \kappa_s(g_m(\rho_m^*, \rho_s^*), g_s(\rho_m^*, \rho_s^*))$. On the other hand, Assumption 1 implies $\kappa_s(g_m(\rho'_m, \rho'_s), g_s(\rho'_m, \rho'_s)) \leq \kappa_s(g_m(\rho_m^*, \rho_s^*), g_s(\rho_m^*, \rho_s^*))$, which is a contradiction. This completes the proof. \blacksquare

Proof of Proposition 3. We now proceed to proof part (2) of the Proposition. For any $\rho_m \in \left[0, \frac{\gamma}{2\tau_u} \right]$, define $\rho_s(\rho_m; \eta)$ as the solution to $\rho_s = f_s(g_s(\rho_m, \rho_s); \eta)$. From Lemma 2 we have that $f_s(g_s(\rho_m, \rho_s); \eta)$ is strictly decreasing in ρ_s . Furthermore, for sufficiently high information production costs, we have $f_s(g_s(\rho_m, 0); \eta) < \frac{\gamma}{2\tau_u}$. The function $\rho_s(\rho_m; \eta)$ is therefore uniquely defined. The function $\rho_s(\rho_m; \eta)$ is also increasing in η and ρ_m .

In equilibrium we have

$$\rho_m^* = f_m(g_m(\rho_m^*, \rho_s(\rho_m^*; \eta^*))).$$

From the uniqueness result of Proposition 2 we have that for all $\rho_m \in [0, \rho_m^*)$

$$f_m(g_m(\rho_m, \rho_s(\rho_m; \eta^*))) - \rho_m > 0.$$

From Lemma 2 we have

$$f_m(g_m(\rho_m, \rho_s(\rho_m; \eta'))) - \rho_m > 0 \geq f_m(g_m(\rho_m, \rho_s(\rho_m; \eta^*))) - \rho_m$$

with strict inequality if $\rho_s(\rho_m; \eta) > 0$. It follows that $\rho'_m \geq \rho_m^*$, with strict inequality if $\rho_m^* > 0$, and that $\rho'_s \geq \rho_s^*$. \blacksquare

Proof of Proposition 4. Analogous to (23) in the proof of Proposition 1, the expected utility of an indexer, given his information set at the time of trading, is given by

$$\begin{aligned} \mathbb{E}[-\exp(-\gamma W_i) | \mathcal{F}_i] &= \mathbb{E}\left[-\exp\left(-\gamma\left(\sqrt{2}\bar{s}P_m + e_m P_m + \frac{1}{2} \frac{\mathbb{E}[X_m - P_m | \mathcal{F}_i]^2}{\gamma \text{var}(X_m | \mathcal{F}_i)} + e_s X_s\right)\right) | \mathcal{F}_i\right] \\ &= \mathbb{E}\left[-\exp\left(-\gamma\left(\sqrt{2}\bar{s}P_m + e_m P_m + \frac{1}{2} \frac{\mathbb{E}[X_m - P_m | \mathcal{F}_i]^2}{\gamma \text{var}(X_m | \mathcal{F}_i)}\right)\right) | \mathcal{F}_i\right] \mathbb{E}[\exp(-\gamma e_s X_s) | \mathcal{F}_i] \end{aligned}$$

where the equality follows from the independence of (e_s, X_s) and (e_m, P_m, X_m) .

In order to evaluate the ex-ante expected utility, we first find the expected utility conditional on the endowment shock received by the agent. We first note that

$$\mathbb{E}\left[-\exp\left(-\gamma\left(\sqrt{2}\bar{s}P_m + e_m P_m + \frac{1}{2} \frac{\mathbb{E}[X_m - P_m | \mathcal{F}_i]^2}{\gamma \text{var}(X_m | \mathcal{F}_i)}\right)\right) | e_i\right] = \mathbb{E}[-\exp(B^\top V + V^\top AV)] \quad (32)$$

with

$$V = \begin{pmatrix} \frac{\mathbb{E}[X_m - P_m | \mathcal{F}_i]}{\text{var}(X_m | \mathcal{F}_i)} \\ P_m \end{pmatrix}; \quad B = \begin{pmatrix} 0 \\ -\gamma(\sqrt{2}\bar{s} + e_{im}) \end{pmatrix}; \quad A = \begin{pmatrix} -\frac{1}{2}\text{var}(X_m | P_m, e_{im}) & 0 \\ 0 & 0 \end{pmatrix}.$$

The first two moments of V are

$$\mu = \begin{pmatrix} \frac{\mathbb{E}[X_m - P_m | e_i]}{\text{var}(X_m | \mathcal{F}_i)} \\ \mathbb{E}[P_m | e_i] \end{pmatrix}; \quad \Sigma = \begin{pmatrix} \frac{\text{var}(X_m - P_m | e_i) - \text{var}(X_m | \mathcal{F}_i)}{\text{var}(X_m | \mathcal{F}_i)^2} & \frac{\text{cov}(X_m - P_m, P_m | e_i)}{\text{var}(X_m | \mathcal{F}_i)} \\ \frac{\text{cov}(X_m - P_m, P_m | e_i)}{\text{var}(X_m | \mathcal{F}_i)} & \text{var}(P_m | e_i) \end{pmatrix}; \quad (33)$$

where in the last expression we have used the law of total variance-covariance.

Simple algebraic calculations show that

$$|I - 2\Sigma A| = \frac{\text{var}(X_m - P_m|e_i)}{\text{var}(X_m|\mathcal{F}_i)} \quad (34)$$

and similarly

$$(I - 2\Sigma A)^{-1}\Sigma = \begin{pmatrix} \text{var}(X_m|\mathcal{F}_i)^{-1} - \text{var}(X_m - P_m|e_i)^{-1} & \frac{\text{cov}(X_m - P_m, P_m|e_i)}{\text{var}(X_m|e_i)} \\ \frac{\text{cov}(X_m - P_m, P_m|e_i)}{\text{var}(X_m|e_i)} & \text{var}(P_m|e_i) - \frac{\text{cov}(X_m - P_m, P_m|e_i)^2}{\text{var}(X_m - P_m|e_i)} \end{pmatrix}; \quad (35)$$

We next argue that the determinant in (34) is decreasing in ρ_m . By the projection theorem, it is straightforward to check that

$$\frac{\text{var}(X_m - P_m|e_i)}{\text{var}(X_m - P_m|\mathcal{F}_u)} = (\tau_x + \rho_m^2(\tau_z + \tau_u)) \left((1 - b)^2 \tau_x^{-1} + d^2 (\tau_z + \tau_u)^{-1} \right)$$

From Proposition 1 (and in particular (21) in the proof), we can write

$$1 - b = \frac{\tau_x}{\tau_x + \rho_m^2 \tau_z + \gamma \rho_m}; \quad d = \frac{\rho_m \tau_z + \gamma}{\tau_x + \rho_m^2 \tau_z + \gamma \rho_m}, \quad (36)$$

so that

$$\begin{aligned} \frac{\text{var}(X_m - P_m|e_i)}{\text{var}(X_m - P_m|\mathcal{F}_u)} &= \frac{\tau_x + \rho_m^2(\tau_z + \tau_u)}{(\tau_x + \rho_m^2 \tau_z + \gamma \rho_m)^2} (\tau_x + (\rho_m \tau_z + \gamma)^2 (\tau_z + \tau_u)^{-1}) \\ &= \frac{\tau_x^2 + \tau_x(\rho_m \tau_z + \gamma)^2 (\tau_z + \tau_u)^{-1} + \tau_x \rho_m^2 (\tau_z + \tau_u) + \rho_m^2 (\rho_m \tau_z + \gamma)^2}{(\tau_x + \rho_m^2 \tau_z + \gamma \rho_m)^2} \\ &= 1 + \frac{\tau_x}{\tau_z + \tau_u} \frac{(\gamma - \rho_m \tau_u)^2}{(\tau_x + \rho_m^2 \tau_z + \gamma \rho_m)^2}, \end{aligned}$$

where the final equality follows from simple algebraic manipulations. From Proposition 1 we know $\frac{\gamma}{2} - \rho_m \tau_u > 0$, and so certainly $\gamma - \rho_m \tau_u > 0$. Hence the ratio $\frac{\text{var}(X_m - P_m|e_i)}{\text{var}(X_m - P_m|\mathcal{F}_u)}$ is decreasing in ρ_m .

Further tedious calculations show that the term in the exponent can be expressed as

$$\begin{aligned} &b^\top \mu + \mu^\top A \mu + \frac{1}{2} (b + 2A\mu)^\top (I - 2\Sigma A)^{-1} \Sigma (b + 2A\mu) \\ &= -\frac{\text{var}(X_m - P_m|e_{im})}{2} \left(\frac{\mathbb{E}[X_m - P_m|e_{im}]}{\text{var}(X_m - P_m|e_{im})} - \gamma \left(\sqrt{2\bar{s}} + e_{im} \right) \frac{\text{cov}(X_m - P_m, X_m|e_{im})}{\text{var}(X_m - P_m|e_{im})} \right)^2 \\ &\quad - \gamma \left(\left(\sqrt{2\bar{s}} + e_{im} \right) \mathbb{E}[X_m] - \frac{\gamma}{2} \left(\sqrt{2\bar{s}} + e_{im} \right)^2 \text{var}(X_m) \right). \end{aligned}$$

Using (10), and the fact that $\text{cov}(X_m - P_m, X_m|e_{im}) = \text{cov}(X_m - P_m, X_m)$, we have

$$\mathbb{E}[X_m - P_m|e_{im}] - \gamma \left(\sqrt{2}\bar{s} + e_{im} \right) \text{cov}(X_m - P_m, X_m|e_{im}) \quad (37)$$

$$= \mathbb{E}[X_m - P_m|e_{im}] - \gamma \left(\sqrt{2}\bar{s} + e_{im} \right) \frac{\mathbb{E}[X_m - P_m]}{\sqrt{2}\gamma\bar{s}} \quad (38)$$

$$= \left(\frac{\text{cov}(X_m - P_m, e_{im})}{\text{var}(e_{im})} - \gamma \text{cov}(X_m - P_m, X_m) \right) e_{im} \quad (39)$$

We next show that the term in the exponent is decreasing in ρ . From the above formulae, it is sufficient to show that the following function is decreasing in ρ_m :

$$\left(\frac{\text{cov}(X_m - P_m, e_{im})}{\text{var}(e_{im})} - \gamma \text{cov}(X_m - P_m, X_m) \right)^2 \frac{1}{\text{var}(X_m - P_m|e_{im})} = \frac{\left(\frac{d}{(1-b)} \frac{\text{var}(Z_m|e_{im})}{\text{var}(u_{im})} - \gamma\tau_x^{-1} \right)^2}{\tau_x^{-1} + \frac{d^2}{(1-b)^2} \text{var}(Z_m|e_{im})}, \quad (40)$$

where the equality follows from

$$\frac{\text{var}(Z_m)}{\text{var}(e_{im})} = \frac{\tau_u}{\tau_z + \tau_u} = \frac{\text{var}(Z_m|e_{im})}{\text{var}(u_{im})}. \quad (41)$$

From (36) we know $d/(1-b) = (\rho_m\tau_z + \gamma)/\tau_x$, so that we simply have to prove that (40) is decreasing in $d/(1-b)$. Taking derivatives in (40), it is easy to check that the function is decreasing in $d/(1-b)$ if and only if

$$\frac{(\rho_m\tau_z + \gamma) \text{var}(Z_m|e_{im})}{\tau_x \text{var}(u_{im})} < \frac{\gamma}{\tau_x},$$

which indeed holds since, from Proposition 1, $\rho_m \leq \frac{\gamma}{\tau_u}$. ■

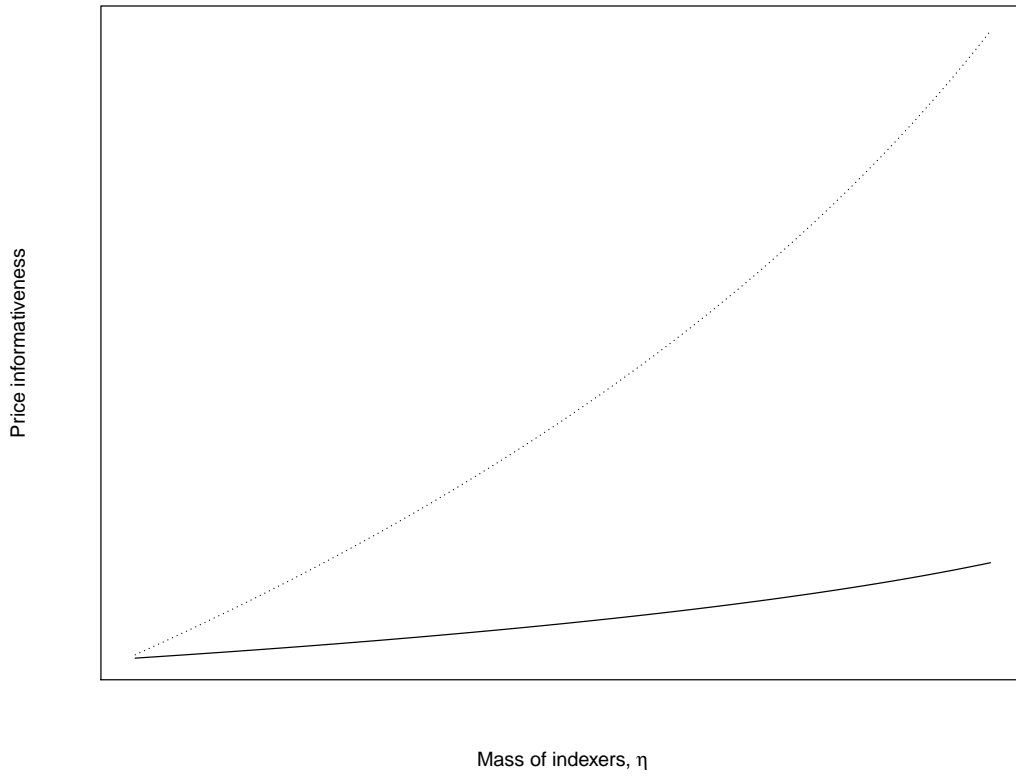


Figure 1: The graphs give the relationship between the price informativeness, $\text{var}(X_j|P_m, P_s)$, for $j = m, s$, and the amount of indexers in our model, η . The solid line depicts the informativeness of the market asset, while the dotted line presents the informativeness of the spread asset.

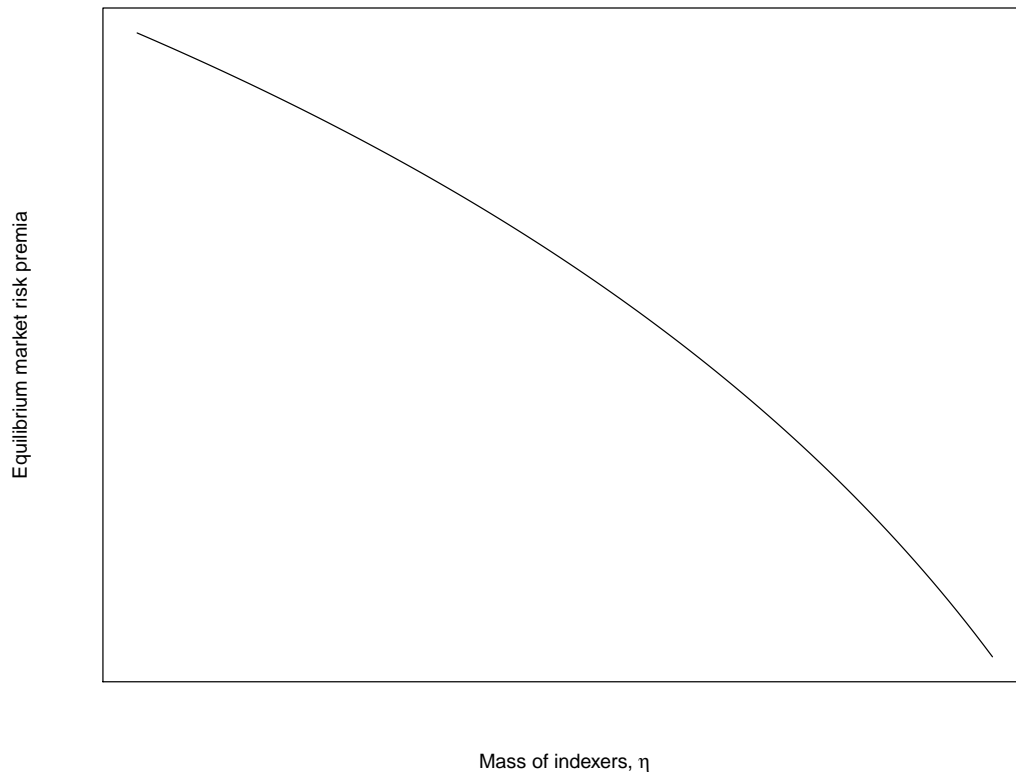


Figure 2: The graph gives the relationship between the equilibrium market risk premium in our model and the amount of indexers, η .

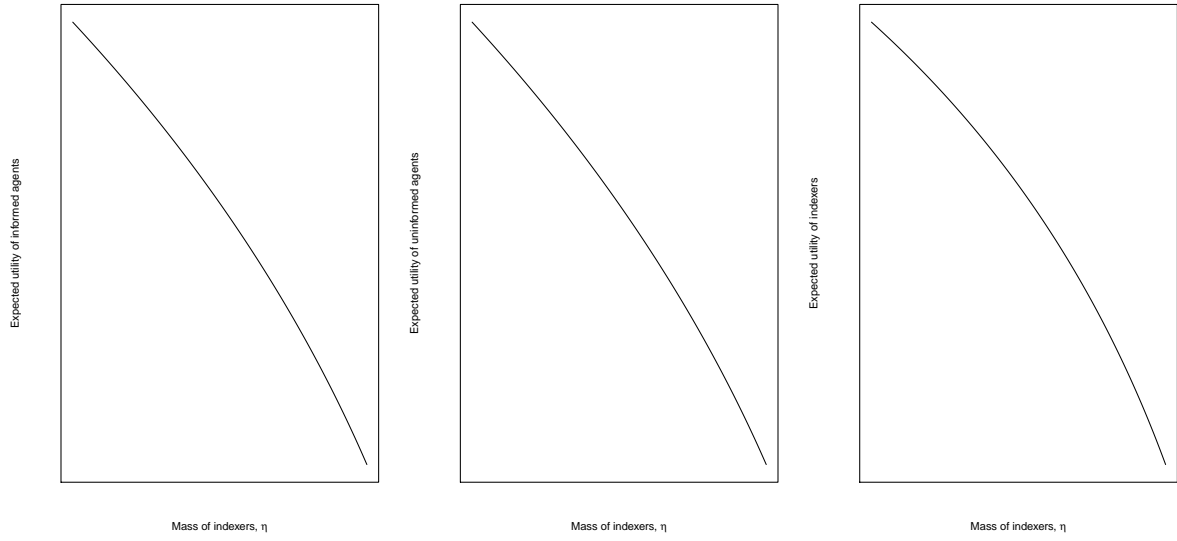


Figure 3: The graph gives the relationship between the expected utility of informed agents (left panel), uninformed active agents (middle panel), and indexers (right panel) as a function of the amount of indexers, η .

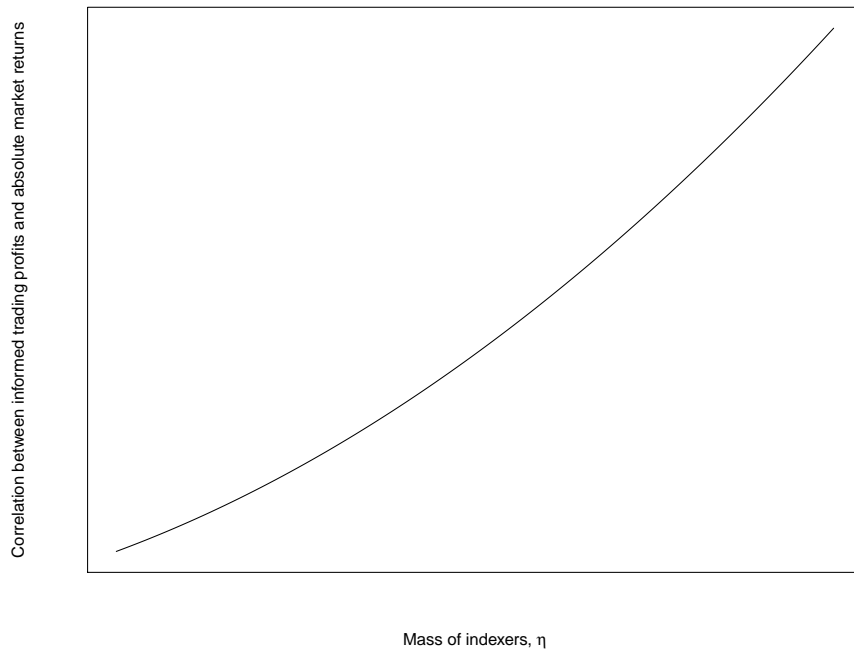


Figure 4: The graph plots the relationship between the correlation of the trading profits of informed traders and market returns, as a function of the amount of indexers, η .