Direct vs. Indirect Federal Bond Subsidies: New Evidence on Cost of Capital

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Abstract

The longstanding debate surrounding the most effective way for the US federal government to subsidize state and local government capital raising received renewed attention in recent years due to the passage and subsequent expiration of the taxable Build America Bond (BAB) program. Recent academic studies, as well as reports from the US Treasury Department, claim that the direct subsidy approach as evidenced by the BAB program provides greater bond borrowing cost benefits to state and local governments compared to traditional tax-exempt bonds. This research investigates the extent to which such borrowing cost benefits may be overstated since it appears previous studies did not adequately account for the early call optionality of tax-exempt bonds.

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Introduction

The longstanding debate surrounding the most effective way for the U.S. federal government to subsidize state and local government capital activities received renewed attention in recent years due to the passage and subsequent expiration of the taxable Build America Bond (BAB) program. This debate has revolved around direct versus indirect subsidies of cost of capital. Traditional tax-exempt municipal bonds represent the indirect approach while taxable BABs are an example of the direct approach. Recent academic studies, as well as reports from the U.S. Treasury Department, claim that the direct subsidy approach as evidenced by the BAB program provides greater bond borrowing cost benefits to state and local governments compared to traditional tax-exempt bonds.

This research investigates the extent to which previous research may have overstated such borrowing cost benefits since it appears such studies did not adequately account for differences in call optionality between the two types of bonds, traditional tax-exempts and BABs. These differences in call optionality have direct implications for the likelihood that state and local governments will be able to refinance their debt in the future to reduce the overall cost of capital on their financings.[1] Unfortunately, failure to take into account call optionality is often common in studies performed by academics and other public policy researchers.[2] Through matched pair analyses of 43 BAB and tax-exempt bond issues sold in California, this paper utilizes refunding adjusted yield (RAY), a novel but more accurate estimate of tax-exempt bond capital cost, to better estimate the capital cost differences between the two subsidy approaches.

Given other concerns related to the direct subsidy approach, a more precise estimate of the benefit

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[1] In the parlance of the municipal bond market, a refinancing is referred to as a "refunding." While there are different types of refundings (advance and current) that have tax policy implications, we will use the terms "refunding" and "refinancing" interchangeably in this paper.

[2] For example, the Brookings Institution released a study in 2016 that found the federal government gave up $3.2 billion in tax revenues by allowing governments to finance sports stadiums on a tax-exempt basis. Brookings estimated the $3.2 billion estimate based on the interest rate spread between tax-exempt and taxable bonds. George Friedlander, a prominent municipal bond market expert, criticized the study on several grounds in *The Bond Buyer*. Germane to our study, Friedlander stated one of his criticisms: "The call right has to be priced in." *The Bond Buyer* authors immediately followed that quote with additional context: "The study looks at 20-year or longer munis and corporates. Corporates typically have "make whole calls," which discourage issuers from calling them. Munis, in contrast have 10-year call dates, and issuers often benefit from calling the bonds" (Hume and Fallor, 2016).

of the direct subsidy approach is valuable as the debate on federal bond subsidies will likely continue given the sizeable capital needs of state and local governments and the ongoing scarcity of federal budgetary resources. This research is especially salient from a federal infrastructure financing policy perspective. During the 2016 presidential campaign, candidate Trump repeatedly suggested a tax credit approach to financing new infrastructure. His advisors also expressed support for taxable direct-subsidy bonds, such as BABs, as an important component of the financing mix for new infrastructure funding. For example, a report released by candidate Trump's senior policy advisors Wilbur Ross and Peter Navarro in October 2016 clearly expressed the incoming administration's acknowledgement of the importance of existing bond methods to finance infrastructure as well as its preference for taxable direct subsidy bonds rather than indirect subsidy tax-exempt bonds:

> "We believe that this tax credit-assisted program could help finance up to a trillion dollars' worth of projects over a ten-year period. This innovative financing option would serve as a critical supplement to existing financing programs, public-private partnerships, Build America Bonds, and other prudent funding opportunities." (page 6)
> "In fact, both Trump and Clinton support the concept of the Build America Bond program. It permits government issuers to issue taxable bonds instead of the usual tax-exempt ones. This provides issuers with access to investors who seek taxable income and therefore broadens the potential market." (page 7)

Many infrastructure experts understand that President Trump's tax credit approach is not feasible for many capital projects since tax credits are irrelevant for projects that do not generate revenue (Forsyth, 2016; Moser, 2016). In this sense, traditional debt finance, subsidized either directly or indirectly by the federal government, will continue to represent the primary financing mechanism for many infrastructure projects. Given the likelihood that traditional debt finance will remain an important component of infrastructure funding, this study analyzes the preferred position proffered by current administration policy makers and many economists' (direct subsidy taxable bonds) compared to the current approach favored by many state and local government (indirect subsidy tax-exempt bonds).[3] In addition to better estimating

[3] The Trump administration released its infrastructure-funding plan on February 11, 2018. While the plan did not include direct subsidy bonds, it did include an expansion of indirect subsidy private activity bonds and incentives to include more participation from the private sector in the funding and financing of the nation's infrastructure (Hume, 2018). Given the Obama administration's and some in the Trump administration's support for the direct subsidy

capital cost differences between the two subsidy approaches, this research also sheds light on the minimum direct subsidy rate required to induce state and local governments to sell taxable direct subsidy bonds (like BABs) rather than traditional tax-exempt bonds.[4]

The paper proceeds as follows. First, we provide an overview of the mechanics of taxable direct subsidy bonds and traditional tax-exempt indirect subsidy bonds, as well as a description of the BAB program. Second, we position this research relative to previous studies that explore the interest cost differences between the indirect and direct subsidy bond approaches. Third, we detail our data and methodology in estimating the benefits of the direct subsidy approach including a description of RAY. This section also identifies some methodological limitations to our study. Fourth, we describe the empirical findings of our analysis. Finally, we discuss these findings in the context of potential federal budgetary policy related to subsidizing state and local government capital activities.

1. Direct and Indirect Subsidy Bonds

The federal government has historically provided capital-financing subsidies to state and local governments in two ways. By far the most prevalent way is the provision of tax exemption on the income received on state and local government bonds, which represents the indirect subsidy approach. Tax exemption lowers the required interest rate demanded by bond buyers vis-à-vis taxable bond alternatives since tax-exempt investors will generally not have to pay taxes on their interest earnings. In this case, buyers make investment decisions based on their individual after-tax (taxable equivalent) yield, which is unique to each individual buyer and determined by their marginal federal tax rate (and state tax rate to the

approach, it is certainly possible that future iterations of this plan or a future one by a different administration will include authorization of direct subsidy bonds (Blinder and Kruger, 2016).

[4] This research evaluates the borrowing cost benefits of taxable direct subsidy bonds versus indirect tax-exempt indirect subsidy bonds. The capital cost of each approach compared to the capital cost of an unsubsidized taxable bond financing by the issuer equals the subsidy benefit. The subsidy cost to the federal government is the loss in foregone tax revenues for the indirect subsidy approach or the cost of the direct subsidy less tax owed on the interest payments for the direct subsidy approach. A specific estimate of the cost to the federal government of each subsidy approach is beyond the scope of this study with the diversity of buyers of tax-exempt bonds and taxable municipal bonds based on marginal tax rates complicating the analysis. However, since a financing's capital cost determines the subsidy benefit for the indirect approach, we can safely claim that the inefficiency of the indirect approach is overstated or understated by the difference between the different capital cost estimates we find in this study (i.e., TIC and RAY).

extent the bond also enjoys state tax exemption). The myriad combinations of tax rates create a fractionalized municipal bond market and makes selling these bonds more difficult. Issuers would optimally want to find as many buyers as possible in the highest marginal tax rates since such investors are willing to pay more for the bond as their individual after tax yields given any coupon would be highest. Thus, federal tax exemption *indirectly* subsidizes state and local government capital activities by facilitating a reduction in the interest rate paid by state and local governments on their tax-exempt bonds. The second approach is to provide the subsidy directly to the issuer of taxable bonds. The direct subsidy approach entails the issuance of taxable bonds by state and local governments with the federal government providing a cash subsidy *directly* to these issuers to offset their interest payments on the bonds. The Build America Bond program represents the largest usage of the direct subsidy approach.

The BAB program was an attempt by the Obama administration to facilitate more cost effective and efficient capital raising by state and local governments in the immediate aftermath of the 2008 / 2009 global financial crisis (GFC). The BAB program was in effect between April 2009 and December 2010. BABs are taxable municipal bonds that entitled the issuing government to receive a 35% direct federal subsidy of the bond interest expense. This direct subsidy aimed to offset the higher interest expense of taxable bonds relative to the tax-exempt issuance alternative. Measured by issuance, the BAB program was a considerable success. As shown in Table 1, state and local governments sold $64 billion and $117 billion in BABs in 2009 and 2010, respectively. This represented 16 percent and 27 percent, respectively, of total municipal bond market issuance in those years. While the BAB program expired on December 31, 2010, there have been several Congressional proposals to reinstate the program with President Obama's support as well as some suggestion by the Trump administration for instituting a similar approach (Schroeder, 2010; Seymour, 2010; West, 2011; Selway, 2012; Puentes and Sabol, 2015; Navarro and Ross, 2016; Blinder and Kruger, 2016).

In addition to encouraging state and local capital investment at a time of a deep economic recession, BABs were also a policy response to actual and perceived shortcomings in the tax-exempt bond market. In some sense, tax-exempt bonds since their creation were more a matter of states' rights than well-thought and precisely designed financing tools for states (and later cities, counties and other local governmental units). Previous research has documented the shortcomings of tax-exempts bonds (Galper and Peterson, 1973; Fortune, 1973; Congressional Budget Office, 2004). Specifically, tax-exempt bonds suffer from several targeting shortcomings including:

1. The benefit of tax-exempt interest income is greatest for those buyers in high marginal tax rates and, thus, wealthier people tend to capture the subsidy at the expense of general federal revenues and other taxpayers. This creates a social equity issue policy makers need to address.
2. The benefit of tax-exempt interest income is valuable solely to United States taxpayers, thus severely limiting the investor base for tax-exempt bonds.
3. The subsidy is maximized only if every bond is sold to the highest marginal beneficiary (those in the highest tax bracket), which is rarely the case. When governments sell their bonds to a mix of low and high tax rate buyers, the rates are set to clear the market for the lowest marginal taxable equivalent yield buyer (i.e., those in the lowest tax bracket). This reduces the amount of available subsidy captured by the issuer and increases their cost of borrowing.

The BAB program aimed to fix all these issues in one fell swoop by creating a homogenous municipal bond market (all BABs would be fully federally taxable). Rather than delivering the subsidy inefficiently and indirectly to bond buyers via the tax code, the federal government delivered the subsidy efficiently and directly to the state or local government via a cash payment.

With the BAB alternative, state and local governments had the option of issuing traditional tax-exempt bonds (and receiving no direct subsidy) or issuing taxable bonds and receiving a 35% subsidy. Mechanically, in the context of these options, issuers and their advisors would estimate (via market polling and comparable bond issuance analysis) the bond yields for both bond types, subtracting the subsidy to arrive at a comparable after-subsidy yield for the BABs. Issuers would choose the approach that resulted in the lowest after-tax yield. Many issuers selected the direct subsidy route, as evidenced in Table 1, since they expected it to be less expensive. Given the recessionary conditions at the time, the scarcity of revenues and low level of interest rates, issuers discounted the value of (and concerns about) losing the refinancing option by issuing BABs instead of traditional tax-exempt bonds, as discussed more fully below. Of course, low market interest rates generally reduce the value of tax exemption also.

Issuers found the taxable / tax-exempt issuance arbitrage attractive and, assuming the federal government had set the subsidy at a level that, in its estimation, was less than the forgone revenue of a similar tax-exempt issuance, all parties are better off, but for the marginal high-tax-bracket tax-exempt bond buyers.[5] That is, the state or local government receives a lower after-subsidy cost of borrowing, taxable bond buyers get access to generally high credit quality long-dated assets with attractive risk-adjusted yields, and the federal government reduces its de facto indirect subsidy by making it directly. Other benefits of the BAB program include a broadening of the market which should tend to drive prices up (yields down) and a reduction of the supply/demand and forward supply calendar constraints which limited issuer flexibility (i.e., the thought being the market could only absorb so much supply so issuers tried to avoid issuing during large issuance supply weeks).

Rate-setting and subsidy targeting inefficiencies may have given rise to the idea of direct subsidy bonds, but despite its perceived inefficiencies, the tax-exempt municipal bond market offered issuers benefits beyond subsidized borrowing rates as determined on the closing date. For state and local governments, if issuers sold their bonds into the broader taxable market, they would need to hew to the conventions of the taxable market to meet taxable bond buyer needs. Historically, the tax-exempt municipal bond market has enjoyed certain unique features generally accepted by tax-exempt buyers, which inured to the benefit of issuers. Chief among those benefits were issuer-favorable optional redemption provisions, most commonly at 10 years at a price of par (i.e., 10-year par call). This stands in contrast to call provisions standard in the taxable market which tend to have a redemption price that makes the investor "whole" given current market conditions (i.e., make-whole call). The make-whole type of redemption is effectively uneconomic to the issuer as it eliminates any potential advantage gained from lower interest rates.

Tax-exempt bond call provisions allowed state and local governments to refinance bonds for

[5] The other policy challenge inherent in direct subsidy bonds is the proper setting of the subsidy. One could claim the proper level should be approximately the average marginal tax rate of the tax-exempt buyer (implied by the ratio of some tax-exempt index to a similar taxable index). Previous research has found this has ranged from 10 and 38% (Atwood, 2003; Poterba and Verdugo, 2008; Congressional Budget Office and Joint Committee on Taxation, 2009; Liu and Denison, 2010; Ang, Bhansali and Xing, 2010b; Longstaff, 2011). Our study provides an estimate of this subsidy rate taking into account the difference in call option between tax-exempt and taxable bonds, which differs from previous studies.

savings on either a current or advanced basis.[6] This call option was highly valuable during the extended period of declining interest rates experienced from the early 1980s until the present. Even absent a declining interest rate environment, state and local governments had grown accustomed to constantly monitoring their bond portfolio for opportunities to refinance maturities, which had "rolled down the yield curve." For example, 20-year bonds issued 12 years ago with a corresponding 20-year coupon were now 8-years to maturity and issuers could refinance them at an 8-year rate, resulting in savings. Many issuers with bond maturities outstanding would evaluate refinancing candidates coincident with new money issuance and select maturities to refund, thus generating incremental savings on the back of new money issuances. For revenue bond issuers in particular, the call flexibility also allowed for low-cost, easy debt restructuring if they encountered financial distress or needed unexpected bond covenant relief. In contrast, the make-whole provisions of BABs did not allow state and local governments to capture these refinancing benefits and made restructuring more costly.

In many respects, issuers (with encouragement from financial advisors and investment bankers) viewed a new money bond issue as the first chapter in the life of the financing. State and local governments often call their tax-exempt bonds prior to maturity either to refinance debt for interest savings, restructure debt or to retire debt early from excess revenues or unspent bond proceeds. Thus, issuers became habituated to issuing callable bonds with the intent to retire them prior to their stated maturity and replace them with new bonds (possibly even extending the original maturity – often referred to a "scoop and toss" restructuring in industry parlance). Thus, for many projects a "lifecycle" or permanent financing was comprised of an initial issuance and subsequent refunding issuances (limited only by the IRS restrictions on refundings) until the entire principal amount was retired.

Indeed, so expected was the future refunding, that investors who demanded premium coupons (to reduce duration) created an incentive for refunding because many callable bonds were issued with coupons,

[6] "Current refundings" are refinancings of bonds no more than 90 days before the call date. "Advance refundings" are refinancings of bonds more than 90 days from the call date. The Tax Cuts and Jobs Act of 2017 prohibited the use of advance refundings as of December 31, 2017.

which resulted in a deep-in-the-money call option embedded in the issue upon issuance. Failing to refund such a bond gave the buyer a higher than expected return (referred to as a "kick yield") and exposed the issuer to potential criticism. The fact that such a multiple financing strategy creates more bond issuance fees makes it popular amongst municipal bond advisors, insurers, underwriters, rating agencies, bond lawyers and the like.[7]

Another downside of the direct subsidy approach is exposure to the federal appropriation process. While some state and local governments may have downplayed such risk, BAB issuers later learned that a federal budget sequester could diminish the subsidy. The Budget Control Act of 2011 resulted in a budget sequester between 2013 and 2020 that reduced BAB subsidies in those years (i.e., 8.70% in 2013, 7.20% in 2014, 7.30% in 2015, 6.80% in 2016, 6.90% in 2017 and 6.60% in 2018, 6.20% in 2019, 5.90% in 2020). Such withholding resulted in issuers being obligated to make the larger taxable payments to bondholders without the expected compensating benefit of having the full subsidy. A recent study of BAB issuers in Illinois estimated a loss of almost $54 million in BAB subsidies between 2013 and 2017 (Luby, 2017). This left some issuers struggling to make debt service payments. For the purposes of this study, failure to consider such sequester reductions overstates the borrowing benefit of BABs. In sum, lack of optional redemption flexibility and sequester risk are two significant drawbacks to direct subsidy bonds. Unlike previous studies, this research aims to incorporate these factors in better estimating the interest cost benefits of the direct subsidy approach compared to the indirect approach.

2. Previous Research

As discussed in the previous section, the inefficiency of the indirect subsidy approach in subsidizing state and local government capital activities has been the focus of considerable research going back as far

[7] One other dynamic created by advance refundings – a specific implementation of this multiple issuance approach which is no longer permitted – is that for a period of time there are two series of tax-exempt bonds outstanding. This results in two subsidies for the same municipal infrastructure project. The Tax Reform Act of 1986 severely limited such transactions and, as previously mentioned, the Tax Cuts and Jobs Act of 2017 completely prohibited them. The lack of economic call options inherent in BABs precludes this double subsidy from overlapping issuing for the same project.

as several decades and continuing to the present (Galper and Peterson, 1973; Fortune, 1973; Congressional Budget Office, 2004; Bond Market Association, 2004; Miller, 2009). However, it was not until the creation of the BAB program that researchers could empirically investigate the two subsidy approaches with a large enough dataset. As shown in Table 1, the issuance of taxable direct subsidy BABs represented 16 and 27 percent of the entire municipal bond market in 2009 and 2010, respectively - a significant share municipal bond issuance.

The U.S. Treasury Department completed the first analysis of the BAB program. It used a similar methodology as ours in that it considered 92 paired issues (i.e., state and local governments that issued BABs and tax-exempt bonds on the same day) between April 2009 and September 2009 (U.S. Department of the Treasury, 2010). For these matched pairs of issues, the U.S. Treasury Department found that issuers of BABs saved on average 31 basis points on their 10-year maturities and 112 basis points on their 30-year bonds compared to traditional tax-exempt bonds. The U.S. Treasury Department updated their analysis in 2011 by expanding the universe of matched pair issues to 528 for bond sales between April 2009 and December 2010 (U.S. Department of the Treasury, 2011). In this report, the U.S. Treasury Department found that issuers realized on average an 84 basis point savings on 30-year BAB maturities compared to tax-exempt bonds and realized significant savings on shorter BAB maturities as well. The U.S. Treasury Department's analyses are fundamentally different than ours in that they simply included call features as a dummy control variable in its regression model to "control" for this difference. However, this does not differentiate between make-whole and par call provisions, which have significantly different implications for the likelihood of refinancing bonds in the future. Call provisions would also manifest themselves in rate differentials on the bonds. On the contrary, our analysis differentiates between call option types and calculates what the tax-exempt cost of capital would be if the call option was actually exercised.

Ang et al. (2010a) found that BABs saved issuers 54 basis points over what the issuer would have paid if it sold traditional tax-exempt bonds. Ang et al.'s methodology is different from ours in several ways but most relevant to our study is that they used a sample of bonds that only included "straight" bonds to avoid "dealing with the computational challenges of valuing call options" (Ang et al., 2010a). Luby (2012) analyzed a matched pair of two bond issues sold by the State of Ohio considering the differences in underwriting costs between BABs and traditional tax-exempt bonds. He found that the BABs provided a benefit relative to tax-exempt bonds, on a yield to maturity basis, of between 6 and 60 basis points depending on the maturity date of the bond (Luby, 2012). However, this analysis only looked at the yield to maturity, relied on a willingness to pay methodology and did not consider any call option differences, if any, between the bonds. Luby calculated the subsidy rate whereby State of Ohio would be indifferent between selling taxable BABs and traditional tax-exempt bonds was 24 percent.

Liu and Denison (2014) represent the most recent empirical analysis of the cost-benefit of the Build America Bond program. A subset of their analysis is like ours in that it looks at a matched pair of BAB and tax-exempt issues sold in California between April 2009 and December 2010. Their matched pair includes 53 BAB bond issues and 64 traditional tax-exempt bond issues sold in California by 48 different governments on the same day. Like our study, Liu and Denison (2014) calculate the true interest cost (TIC) on each bond issue, BAB and traditional tax-exempt. The use of TIC in measuring capital costs is different from all the previous studies mentioned above. Liu and Denison found that BABs have on average a 65 basis point lower TIC than traditional tax-exempts after controlling for other specific bond issue differences, including callability. They found that the subsidy rate whereby the matched pair California issuers would be indifferent between selling taxable BABs and traditional tax-exempt bonds (what they call the "implied tax rate") was 25 percent, similar to 24 percent estimated in Luby (2012). Like the U.S. Treasury Department's analyses, Liu and Denison simply include call features as a control variable in their regression model whereas our analysis compares the actual TIC of a BAB issue to a "counterfactual" traditional tax-exempt bond issue TIC (what we call RAY) assuming the tax-exempt bond issue's call options are exercised at some point in the future.

3. Methodology

The BAB program lends itself to the benefits of a matched pair analysis not often available to researchers of financial instruments. In many cases, state and local governments issued traditional tax-exempt bonds and BABs on the same day during the BAB program's life. As described in the previous section, according to Liu and Denison (2014), 48 different governments issued 64 tax-exempt bonds and 53 BABs on the same day in California. Liu and Denison (2014) used these California matched bond pairs to analyze the potential benefits of the BAB program. The benefit of this type of matched pair analysis is that it mitigates many of the challenges to comparing borrowing costs between different bond issues and issuers. Since the same issuer sells the matched bond issue pairs on the same day, the only material differences are the bond's tax status (taxable BAB or traditional tax-exempt) and possibly the maturity dates of individual bonds within the issue.

Previous research on bond borrowing costs, including Liu and Denison's 2014 paper on the BAB program, has used the true interest cost (TIC) to measure the capital cost of the bond issues. TIC is essentially the internal rate of return of a bond issue assuming the bonds are paid to maturity. However, one of the distinguishing features of tax-exempt bonds is that issuers often sell them with a call feature that allows the government to call the bonds early at par. Such an early par call allows for the refinancing of these bonds for interest cost savings. This is in contrast with most taxable securities (including most BABs) that only allow early redemptions at a 'make whole call', which reduces the ability to realize interest cost savings. Thus, in this context, using TIC to estimate capital cost on taxable BABs would be appropriate given the refinancing unlikelihood of these bonds. However, using TIC on tax-exempt bonds would *not* be appropriate, given their refinancing likelihood, and thus would overstate their capital cost. An alternative cost of capital metric known as RAY, refunding adjusted yield, addresses the problematic assumption of TIC that issuers will pay the debt service until maturity for tax-exempt bond issues (Orr and Luby, 2019). RAY incorporates the possibility that a municipal borrower will refinance a new municipal security offering sometime in the future based on a realistic modeling of future bond refinancings.

This research analyzes 43 matched pairs of California bonds (BABs and tax-exempts) issued in 2009 and 2010. We gathered these matched pairs of bonds by examining the U.S. Treasury Department's listing of BABs by state and reading the offering documents (official statement) for each California BAB issue to see if the issuer also sold a traditional tax-exempt issue as part of that issue. From this crossexamination, we found 52 bond issues (similar to Liu and Denison). However, eight of these issues were tax-exempt capital appreciation bonds, which RAY cannot calculate.[8] Thus, we believe the difference in our number of matched pair BABs in California (43) and Liu and Denison's number (53) is mainly due to their inclusion of matched pairs that included capital appreciation bonds. These 43 matched pairs include only BABs that have make-whole calls since the lack of callability of BABs compared to traditional tax-exempts is the primary focus of this study. For capital cost calculation purposes, we gathered data in each bond issue's official statement (retrieved from the MSRB's EMMA database) and municipal bond indices data from Bloomberg.

We take the following steps in our analysis for each matched pair. First, we calculate two TICs for the actual BAB issue: a) the TIC ignoring the 35% federal direct subsidy and b) the TIC incorporating the 35% direct subsidy.[9] Second, we calculate the TIC and RAY on the BAB issue assuming the government sold tax-exempts using the pricing of the tax-exempt bonds actually sold by these issuers on the same day.[10] This essentially represents the counterfactual of the issuers selling tax-exempt bonds rather than a taxable BAB. A confounding element to the calculation of the counterfactual capital cost is that for most matched pairs there was not complete overlap in terms of the maturity dates of the actual BABs sold and the actual tax-exempt bonds sold. For example, in many cases, the tax-exempt bonds had shorter maturities than the BABs did.[11] In these cases, we cannot directly observe what the tax-exempt yields would be for the BABs if the BABs were sold tax-exempt. In the event of limited or no overlap, we construct the counterfactual tax-exempt yield curve in the following manner: a) we observe the spread of the longest dated actual tax-exempt bond to Bloomberg's AAA tax-exempt municipal bond index for that maturity and b) we add that

[8] Because of the significant difference in structure and repayment terms, capital appreciation bonds and current interest bonds are not easily comparable in terms of bond prices/yields.
[9] All-in-TIC is different than TIC in that all-in-TIC takes into account issuance costs. Our TIC and RAY calculations are essentially all-in-TIC and all-in-RAY since we assume 0.50% for cost of issuance.
[10] We assume issuers would sell the counterfactual bonds with a ten-year par call, the standard call feature in the tax-exempt bond market today.
[11] In general, taxable bond buyers are interested in longer-dates maturities. As such, the primary pricing benefit of taxable versus tax-exempt bonds was on the longer end the curve, which state and local governments exploited in terms of the amortization schedule it used in structuring the tax-exempt and taxable bonds.

spread to the Bloomberg tax-exempt AAA yield for each BAB maturity in which there was no overlap with an actual tax-exempt maturity.

Third, we compare the actual BAB TIC taking into account the 35% subsidy and the counterfactual tax-exempt TIC and RAY to estimate the benefit of the direct subsidy approach (actual BAB) compared to the indirect subsidy approach (counterfactual tax-exempt). Fourth, based on the BAB TIC ignoring the 35% federal subsidy, we then calculate two 'neutral subsidy rates' for each issue: a) the direct subsidy rate at which issuers would have been indifferent between issuing tax-exempt bonds and taxable BABs using TIC for the tax-exempt capital cost and b) the direct subsidy rate at which issuers would have been indifferent between issuing tax-exempt bonds and taxable BABs using RAY for the tax-exempt capital cost. The difference between these two neutral subsidy rates represents the overstatement of benefit of the direct subsidy rate compared to the indirect subsidy rate by using TIC rather than RAY as the capital cost estimate. Table 2 provides a summary of the four steps assuming a hypothetical matched pair with a bullet maturity BAB and tax-exempt bond.

The Appendix provides a more detailed description of the general approach of the RAY methodology. Orr and Luby (2019) offer a detailed description of the RAY methodology even more specifically. For the more casual reader, RAY is the yield that recovers the market price from the average of simulated debt service adjusted for future refunding activity. We calculate RAY assuming 5,000 simulations with current refundings executed based on the opportunity index criteria used by many governments.[12] Given issuers are essentially holding unhedged call options embedded in their bonds, it is appropriate to use a real-world market model for yield curve simulations since standard bond option "pricing" models would not be appropriate in this setting (Nawalkha and Rebonato, 2011). This model is particularly well suited to municipal call options / refunding analysis because it allows us to capture the dynamics of both the tax-exempt borrower's and taxable escrow markets simultaneously. While some of

[12] For an example of opportunity cost index criteria in deciding to refund bonds see http://www.tos.ohio.gov/Documents/Investor/OBM-Debt%20and%20Derivative%20Mgt%20Policy-2015%20Final.pdf

the assumptions of the RAY approach are different than other option adjusted spread (OAS) models (Kalotay and May, 1998; Zhang and Li, 2004), the general approach is the same and RAY resides in the general ecosystem of models that consider the value of the option to call the bonds early for interest cost savings in determining its capital cost estimate, which is not accounted for in studies using TIC.

Our study's methodology has some limitations. Because RAY relies on the historical pattern of interest rate environments, the primary limitation of this methodology is that the future may not look like the past. However, this can work in both directions. That is, the future interest rate environment may be more or less conducive to refinancing our sample of bonds for interest cost savings. As such, it is impossible for us to determine if our RAY estimates likely overstate or understate what the actual capital cost will be for the counterfactual tax-exempt approach. Another limitation of the study is its generalizability. Our study essentially represents a relatively small convenience sample of 43 bonds sold in California. In addition, California is a "specialty state" with high state income tax rates that makes tax-exempt California bonds more attractive to in-state investors that may not be applicable to other states. Thus, we are limited in generalizing our findings to the universe of BAB and tax-exempt bonds and therefore one should treat our results as more "illustrative" than "definitive." However, it should be again noted as described above, the matched pair approach offers the benefit of a better control of issuer and market specific factors that may confound other econometric analyses.

4. Results

Table 3 provides detail on the California bond issues used in our analysis. The table arranges the issuance by the dated date of each issue. There were 43 bond issues, which included total par amount of \$12.8 billion in BABs and \$5.2 billion in traditional tax-exempts, sold between June 2009 and December 2010. On an issue-by-issue basis, the BAB issues were generally larger on average (\$300 million) than the tax-exempt bond issues (\$120 million)[13]. The largest BAB and tax-exempt issues were \$1.37 billion sold

[13] The difference in size of the bond issues does pose a modest limitation in terms of the bonds being truly matched pairs. However, we would expect that the larger size BAB issues would have bond yields lower than tax-exempts since they were sold in larger tranches since there is generally more investor demand larger tranches. Thus, our

by Los Angeles Unified School District and $524 million by California State Public Works, respectively. The smallest BAB and tax-exempt issues were $51 million and $5 million, respectively, sold by San Mateo Union High School District. The BABs were longer-dated on average with an average life of 23.6 compared to 10.0 average life for the tax-exempt issues.

Table 4 details calculation of TIC and RAY for each issue (i.e., actual BAB and counterfactual tax-exempt TIC and RAY). Graph 1 provides a visual representation of these three cost of capital estimates for each bond issue. From these calculations, the table and graph show the capital cost benefit (if any) of BABs versus the counterfactual tax-exempt based on separate calculations of TIC and RAY. The average BAB TIC without and with the 35% federal subsidy was 6.530% and 4.261%, respectively. The average counterfactual tax-exempt TIC was 4.904%, which represented a 64 basis point increase over the BAB TIC assuming the full 35% subsidy (4.261%)[14]. While our universe of bonds is slightly different from those Liu and Denison (2014) used, our result is very similar to their finding of a 65 basis point lower BAB TIC than traditional tax-exempts (Liu and Denison, 2014). The average counterfactual tax-exempt RAY was 4.608%, which represented a 35 basis point increase over the BAB TIC assuming the full 35% subsidy[15]. However, using RAY rather than TIC as the capital cost estimate results in a 29 basis point reduction in the benefit of BABs over traditional tax-exempts. Graph 2 illustrates the cost of capital difference under all three scenarios for a specific California bond issue, namely the $486,130,000 Board of Regents of the University of California bonds sold in September 2010. For these bonds, the BAB TIC with the subsidy was 3.905%, the counterfactual tax-exempt TIC was 4.684% and the counterfactual tax-exempt RAY was

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results related to the estimated benefit of BABs over tax-exempts may be slightly overstated due to differences in bond size.

[14] The difference in borrowing cost between the subsidized BABs and their paired counterfactual tax-exempt issues based on TIC is roughly 0.64 percentage points. The T-statistic for the matched pair test (-12.49) is statistically significantly non-zero at the 99 percent level; based on a counterfactual tax-exempt issue using TIC, the subsidized BABs have a cheaper cost of capital.

[15] The difference in borrowing between the subsidized BABs and their paired counterfactual tax-exempt issues based on RAY is roughly 0.35 percentage points. The T-statistic for the matched pair test (-6.37) is statistically significantly non-zero at the 99 percent level; based on a counterfactual tax-exempt issue using RAY, the subsidized BABs have a cheaper cost of capital.

4.261%. For these bonds, using RAY rather than TIC resulted in a 42 basis point reduction in the benefit of BABs over traditional tax-exempt bonds.

Based on this analysis, using TIC instead of RAY in estimating the capital cost of tax-exempt bond alternative results, on average, in a 45% overstatement in the benefit of taxable BABs versus traditional tax-exempts (i.e., ((0.64% - 0.35%) / 0.64%)). Moreover, there were five bond issues wherein the counterfactual tax-exempt RAY was actually lower than the BAB TIC assuming the full 35% subsidy. This represents issues whereby the government would have been better off issuing traditional tax-exempts rather than taxable BABs.

Table 5 details the "neutral subsidy rate" (i.e., the federal subsidy rate whereby the issuer would be indifferent between selling traditional tax-exempt bonds or taxable BABs) for each bond issue based on TIC and RAY. Based on the counterfactual tax-exempt TIC, the average neutral subsidy rate was 24.61%. This is consistent with Luby (2012) and Liu and Denison (2014) who estimated the implied subsidy rates to be 24% and 25%, respectively. However, based on the counterfactual tax-exempt RAY, the average neutral subsidy rate was 29.22%. Thus, based on this analysis, using TIC instead of RAY results in an almost 19% understatement in the neutral subsidy rate (i.e., ((29.22% - 24.61%) / 24.61%)) needed to entice governments to sell taxable BABs instead of traditional tax-exempt bonds.

Tables 4 and 5 estimated the benefit of taxable BABs and the neutral subsidy rate assuming the bond issuers received the full 35% direct subsidy from the federal government. However, there was always a risk that the federal government would reduce their bond subsidies and state and local governments realized such risk as a result of the federal budget sequester that was implemented between 2013 and 2018. Tables 6 and 7 estimate the actual capital cost for the $250 million Metropolitan Water District of Southern California BABs dated August 11, 2009 taking into account budget sequester amounts in fiscal years 2013 through 2018 that reduced federal subsidies. Table 6 compares the actual BAB capital cost to the counterfactual TIC and RAY assuming Metropolitan Water District of Southern California sold tax-exempt bonds. The actual BAB TIC rose from 4.165% to 4.211% taking into accounts the effect of the federal budget sequester. The counterfactual tax-exempt RAY was 4.385% which represented a 22 basis point increase over the expected BAB capital cost but only a 17 basis point increase over the actual BAB capital accounting for the sequester. Table 7 translates the capital cost estimate from TIC into dollars of debt service. The expected benefit of the BABs relative to traditional tax-exempt bonds as measured by RAY was $7.5 million but was reduced to $5.5 million after accounting for the sequester reductions. Table 7 also provides a sense of the difference between RAY and TIC in terms of the estimate of counterfactual cost of capital in debt service terms. The total debt service under TIC (i.e., assuming the issuer does not refinance the bonds) was $506,017,118 while total debt service under RAY (assuming a refinancing) was $469,823,584. This represents a $36,193,534 increase in estimated debt service costs by using tax-exempt TIC rather than RAY.

Table 8 extends the analysis beyond estimating the benefit of BABs based on different capital cost assumptions. Specifically, the table provides estimates of RAY assuming either a current or advance refunding. As previously stated, the Tax Cuts and Jobs Act of 2017 prohibited state and local governments from executing advance refundings. This prohibition aimed to reduce the federal tax expenditure loss from state and local governments having two bond issues outstanding financing the same project for extended periods. However, many market participants believed such advance refunding restriction would increase borrowing costs significantly for state and local governments. This prohibition did not include current refundings, which include bonds whose call date is within 90 days. The analysis provided in the previous tables all assumed current tax law. That is, the prohibition of advance refundings but the allowance of current refundings. Table 8 shows that the average tax-exempt RAY assuming current refundings was 4.608% and 4.590% assuming advance refundings were allowable. This represents a 1.8 basis point increase in capital costs by restricting advance refundings. This results in 0.99% decrease in the neutral subsidy rate from 29.51% to 29.22%

5. Discussion

The results described in the previous section provide a quantification of the overstatement of the capital cost benefit of BABs compared to traditional tax-exempt bonds accounting for differences in call optionality. Our analysis of California issuers who sold BABs and tax-exempt bonds on the same day show that BABs generally represented a lower cost of borrowing for most governments. However, based on our RAY estimates, such benefit is substantially smaller than the findings from previous research. Specifically, by using TIC and thus not considering the likelihood of future refinancing results in a 45% overstatement in the borrowing cost benefit of BABS compared to tax-exempt bonds and a 19% understatement in the neutral subsidy rate. In addition, the benefit of BABs over tax-exempt bonds is not absolute. In fact, a few issuers would have realized interest cost savings by selling traditional tax-exempt bonds compared to BABs based on RAY.

Moreover, except for the BABs sold by the Metropolitan Water District of Southern California, our analysis does not adjust BAB TIC estimates for the budget sequester that reduced federal bond subsidies between 2013 and 2017. As shown in our analysis of the Metropolitan Water District of Southern California, the federal budget sequester reduced the benefit of BABs even further compared to the capital cost accounting for the likelihood of tax-exempt bonds being refinanced in the future as captured in our RAY calculation. While the budget deal reached in February 2018 ended the sequester for some spending, it did not include BAB subsidies. BABs are still exposed to federal budget appropriation risk and still subject to the sequester. In fact, at the end of 2017, there was a concern that the pending tax bills that Congress needed to reconcile would require 100% of BAB subsidies be zeroed out due to PAY-GO requirements under a previous federal budget law passed in 2010 (Hume, 2017).

In terms of implementing a new direct subsidy bond program in the future, federal policymakers should be cognizant of the “burn” many state and local governments felt with the sequester in terms of their BAB subsidies being reduced. Such “burn” was exacerbated when some issuers realized that they could not economically refinance their BABs due to their make-whole call provisions. Policymakers should also be aware of other concerns that many state and local governments have in terms of direct subsidy bonds related to increased administrative burden and concern about loss of fiscal autonomy (Schroeder, 2010; Creswell, 2010; McDonald and Hart, 2010; Luby, 2012). These potential downsides need to be considered with the findings of this study, which shows that BABs are not as beneficial over traditional tax-exempts as prior research suggests after accounting for differences in call optionality between the two types of subsidy bonds.

Taken together, these drawbacks offer strong policy implications for the optimal structure of a direct subsidy bond program. As shown in our neutral subsidy rate estimates, a 35% subsidy rate, like used in the BAB program, is probably overly generous in terms of incentivizing state and local governments to sell taxable direct subsidy bonds rather than traditional tax-exempt bonds. However, it is probably not as generous as previous studies have suggested. That is, while some previous research has shown the implicit tax rate (neutral subsidy rate) in the mid-20s, federal policymakers should be leery of such estimates. Assuming state and local government financial managers are cognizant of the borrowing cost benefit of the 10-year par call included in most tax-exempt bonds, our capital cost estimates provide evidence that a successful taxable direct subsidy bond program would likely need a direct subsidy rate greater than 30%. President Obama proposed the creation of the America Fast Forward (AFF) bond program in 2013 whereby the U.S. Treasury Department would provide a 28% direct subsidy on AFF bonds (Puentes and Sabol, 2015). While the AFF bond program never became law, the estimates in this study cast doubt on whether a 28% subsidy would have incentivized governments sufficiently to use such a program extensively.

Finally, our estimation of the capital cost for our sample of California issues based on current versus advance refunding structures is highly salient given the recently enacted tax reform legislation. The government finance community including issuers, bond dealers, municipal advisors and bond counsels were almost universally against the advance refunding prohibition as ultimately included in the Tax Cut and Jobs Act of 2017 (Hume and Tumulty, 2017). However, our analysis shows that the interest cost benefits for the counterfactual tax-exempt bonds assuming the availability of an advance refunding was existent but relatively small (i.e., 1.8 basis points) compared to issuers waiting to refund their bonds on a current refunding basis. This provides additional support to the importance of systematically valuing the call option and, specifically, to the notion that state and local governments are not necessarily worse off waiting until the call date to refinance their debt (Boyce and Kalotay, 1979; Kalotay and May, 1998; Kalotay, 2007;

Zhang and Li, 2004; Orr and de la Nuez, 2013; Orr and de la Nuez, 2014; Orr and Luby, 2019). Of course, these results are only representative of 43 counterfactual bond issues, so we need to be careful offering strong policy implications based on this finding. In addition, the analysis assumes that the future interest rate environment approximates the past, which, of course, may not be the case. Nevertheless, such results certainly warrant further exploration given the likelihood that the municipal bond community will be advocating in the future for the federal government to repeal or relax this advance refunding restriction.

6. Conclusion

This is the first study to our knowledge that estimates the benefit of the direct bond subsidy approach taking into account differences in the specific call features between taxable and tax-exempt bonds. We believe this study provides a more accurate estimate of the capital cost differences between subsidy approaches. Such estimates offer policy implications for the setting of the optimal subsidy rate assuming the federal government institutes a new direct subsidy bond program in the future. In general, given other considerations of the direct subsidy approach compared to the indirect approach as well as the impact of the recent federal budget sequester on BAB subsidies, federal policymakers likely need to be more aggressive than previously thought in terms of setting a subsidy rate that will induce state and local governments to issue taxable direct subsidy bonds. Of course, a more generous subsidy comes at a cost of a larger budget outlay to the federal government. In addition, there is no guarantee that there will be a substantial "take-up" of a new direct subsidy bond program given some state and local governments' sour taste from the BAB program and their ongoing concern of exposure to the federal appropriation process and resulting sequester risk and the reduced refinancing opportunities afforded by taxable bonds.

7. Disclosures

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9. Tables and Graphs

Table 1, All Build America Bonds (BABs)
BAB Sales by Month, 2009-2010 (par amount in $ billions)

Source: U.S. Treasury Department and Luby 2012

	Build America Bonds	Total U.S. Municipal Bond Issuance	Percent of BABs to Total Municipal Bond Issuance		Build America Bonds	Total U.S. Municipal Bond Issuance	Percent of BABs to Total Municipal Bond Issuance
Jan 2009		23.2	N/A	Jan 2010	7.0	32.7	21.55
Feb 2009		23.4	N/A	Feb 2010	7.1	27.2	26.25
Mar 2009		38.9	N/A	Mar 2010	12.6	44.4	28.41
Apr 2009	7.9	36.8	21.34	Apr 2010	6.5	27.4	23.82
May 2009	2.7	30.4	8.89	May2010	9.3	38.3	24.22
Jun 2009	5.0	43.8	11.47	Jun 2010	9.4	34.7	26.98
Jul 2009	3.6	26.1	13.69	Jul 2010	6.4	29.1	22.15
Aug 2009	9.6	36.4	26.51	Aug 2010	5.4	29.2	18.41
Sep 2009	6.7	30.0	22.43	Sep 2010	9.4	35.5	26.52
Oct 2009	12.9	46.4	27.86	Oct 2010	12.6	45.6	27.65
Nov 2009	7.6	38.3	19.88	Nov 2010	16.1	45.2	35.60
Dec 2009	8.1	36.1	22.34	Dec 2010	15.4	40.9	37.65
2009 Total	**64.1**	**409.7**	**15.65**	**2010 Total**	**117.3**	**430.1**	**27.27**

Table 2, Methodology Summary
Step-by-Step, Hypothetical Example, Bullet Maturity

Step	Action	Result	Calculation
Step 1	Calculate BAB TIC before 35% subsidy	10%	
	Calculate BAB TIC after 35% subsidy	6.5%	(10% * (1-35%))
Step 2	Calculate counterfactual TIC	8.0%	
	Calculate counterfactual RAY	7.5%	

Step 3	BAB benefit over counterfactual tax-exempt (TIC)	1.5%	(8.0% - 6.5%)
	BAB benefit over counterfactual tax-exempt (RAY)	1.0%	(7.5% - 6.5%)
	% overstatement of BAB benefit by using TIC instead of RAY	33.33%	((1.5% - 1.0%) / 1.5%)
Step 4	Neutral subsidy rate based on TIC	20%	1 – (8.0% / 10%)
	Neutral subsidy rate based on RAY	25%	1 – (7.5% /10%)
	% understatement of neutral subsidy rate by using TIC instead of RAY	25%	((25% - 20%) / 20%)

Table 3, California Build America Bonds

California BABs Sold on Same Date as Tax-Exempt Bonds by Same Issuer (par amount in $ millions)

Source: U.S. Treasury Department and bond official statements

Issuer	Dated Date	BAB Par Amount	Tax-Exempt Par Amount
Southern California Metropolitan Water District	6/25/2009	78	22
Tuolumne Wind Project Authority	7/14/2009	152	276
Metropolitan Water District of Southern California	8/11/2009	250	81
Oakland Union School District	8/12/2009	71	88
Pasadena Unified School District	9/17/2009	85	40
City & County of San Francisco	10/7/2009	130	38
Los Angeles Unified School District	10/15/2009	1,370	206
California State Public Works	10/29/2009	250	524
Southwestern Community College	11/5/2009	90	10
Los Angeles Department of Water and Power	12/3/2009	346	141
Los Angeles Department of Airports	12/3/2009	307	246
University of California	12/17/2009	429	95
City of Riverside	12/22/2009	68	32
City of Fresno	2/3/2010	91	67
San Diego County Water Authority	2/4/2010	526	98
Oxnard Financing Authority	2/11/2010	84	16
San Mateo Union High School District	2/17/2010	51	5
Calleguas-Las Virgenes Public Financing Authority	2/24/2010	77	21
Los Angeles Unified School District	3/4/2010	1,251	479
City & County of San Francisco	3/24/2010	209	121

California State University	4/6/2010	205	147
California State Public Works	4/21/2010	246	83
San Francisco Unified School District	5/27/2010	72	100
Los Angeles Department of Water and Power	6/2/2010	616	52
City & County of San Francisco	6/8/2010	193	47
San Francisco City & County Public Utilities	6/17/2010	418	57
Northern California Power Agency	6/24/2010	177	99
Northern California Power Agency	6/24/2010	110	99
San Francisco City & County Public Utilities	8/4/2010	344	103
University of California	9/30/2010	486	196
San Diego County Regional Airport Authority	10/5/2010	215	313
East Bay Municipal Utility District	10/20/2010	150	58
City of Los Angeles	10/21/2010	177	200
Bay Area Toll Authority	11/4/2010	475	410
San Diego County Regional Transportation Commission	11/10/2010	339	11
Los Angeles County Metropolitan Transportation Authority	11/16/2010	574	158
Santa Clara Valley Transportation Authority	11/17/2010	470	176
University of California	11/18/2010	700	48
Riverside County Transportation Commission	11/30/2010	112	38
City of Newport Beach	11/30/2010	107	20
City of Riverside	12/16/2010	133	7
San Francisco City & County Public Utilities	12/22/2010	351	87
Orange County Local Transportation Authority	12/23/2010	294	59
Total		**12,879**	**5,174**
Averages		**300**	**120**

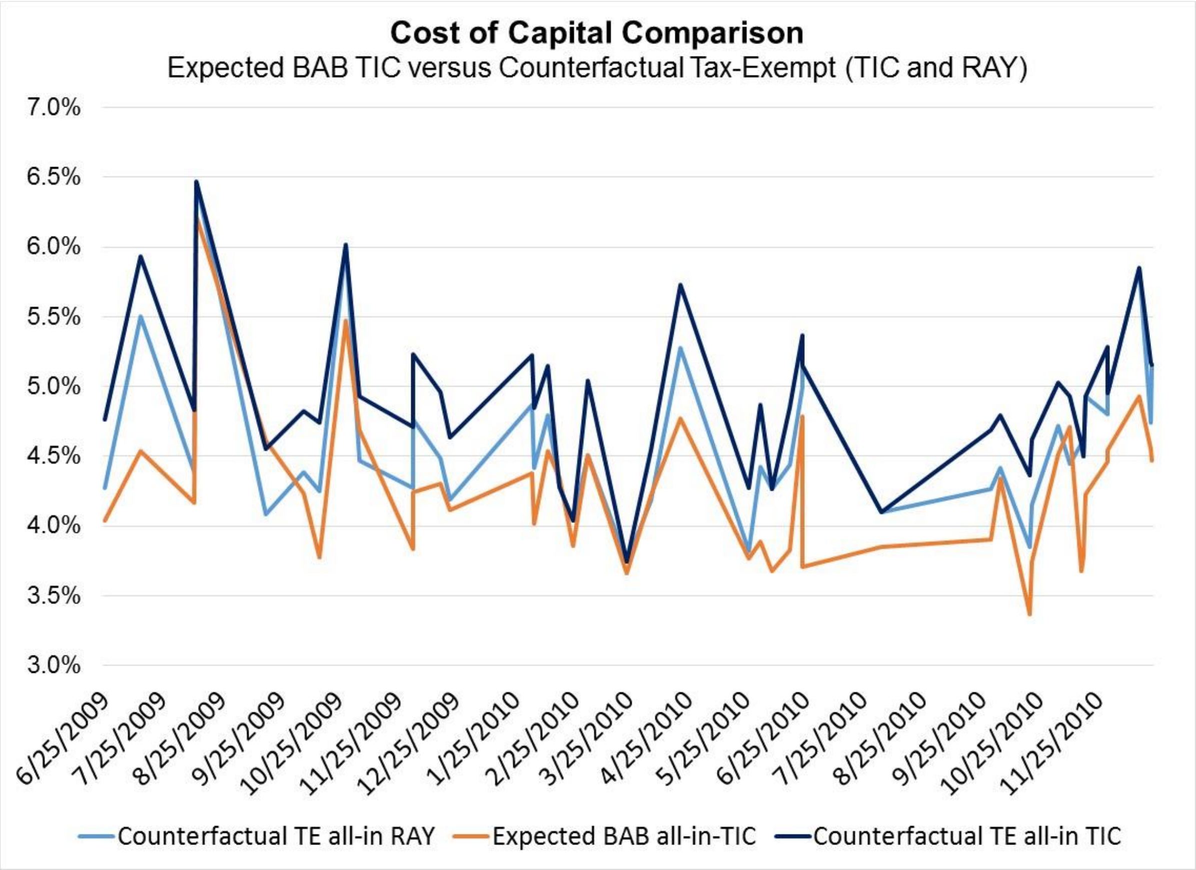
Table 4, California Build America Bonds
California BABs Sold on Same Date as Tax-Exempt Bonds by Same Issuer

Comparison of Actual BAB versus Counterfactual Tax-Exempt (par amount in $ millions)

Issuer	Dated Date	BAB Par Amount	Actual BAB all-in TIC without Subsidy	(A) Actual BAB all-in TIC with Subsidy	(B) Counter-factual Tax-Exempt all-in TIC	(C) Counter-factual Tax-Exempt all-in RAY	Actual BAB all-in TIC with Subsidy minus Counter-factual Tax-Exempt all-in TIC (A) – (B)	Actual BAB all-in TIC with Subsidy minus Counter-factual Tax-Exempt all-in RAY (A) – (C)
Southern California Metropolitan Water District	6/25/2009	78	6.196%	4.038%	4.765%	4.274%	-0.73%	-0.24%
Tuolumne Wind Project Authority	7/14/2009	152	6.963%	4.533%	5.935%	5.505%	-1.40%	-0.97%
Metropolitan Water District of Southern California	8/11/2009	250	6.387%	4.165%	4.832%	4.385%	-0.67%	-0.22%
Oakland Union School District	8/12/2009	71	9.555%	6.216%	6.468%	6.468%	-0.25%	-0.25%
Pasadena Unified School District	9/17/2009	85	7.069%	4.609%	4.554%	4.081%	**0.06%**	**0.53%**
City & County of San Francisco	10/7/2009	130	6.498%	4.231%	4.824%	4.383%	-0.59%	-0.15%
Los Angeles Unified School District	10/15/2009	1,370	5.793%	3.772%	4.740%	4.249%	-0.97%	-0.48%
California State Public Works	10/29/2009	250	8.412%	5.473%	6.018%	6.018%	-0.54%	-0.54%
Southwestern Community College	11/5/2009	90	7.195%	4.686%	4.925%	4.465%	-0.24%	**0.22%**
Los Angeles Department of Water and Power	12/3/2009	346	5.876%	3.835%	4.708%	4.275%	-0.87%	-0.44%
Los Angeles Department of Airports	12/3/2009	307	6.504%	4.243%	5.234%	4.763%	-0.99%	-0.52%
University of California	12/17/2009	429	6.610%	4.302%	4.960%	4.484%	-0.66%	-0.18%
City of Riverside	12/22/2009	68	6.297%	4.114%	4.636%	4.186%	-0.52%	-0.07%
City of Fresno	2/3/2010	91	6.728%	4.381%	5.221%	4.865%	-0.84%	-0.48%
San Diego County Water Authority	2/4/2010	526	6.176%	4.019%	4.845%	4.415%	-0.83%	-0.40%
Oxnard Financing Authority	2/11/2010	84	6.966%	4.535%	5.145%	4.795%	-0.61%	-0.26%
San Mateo Union High School District	2/17/2010	51	6.671%	4.349%	4.278%	4.278%	**0.07%**	**0.07%**
Calleguas-Las Virgenes Public Financing Authority	2/24/2010	77	5.917%	3.856%	4.038%	4.038%	-0.18%	-0.18%
Los Angeles Unified School District	3/4/2010	1,251	6.920%	4.505%	5.041%	4.509%	-0.54%	**0.00%**
City & County of San Francisco	3/24/2010	209	5.608%	3.663%	3.745%	3.745%	-0.08%	-0.08%
California State University	4/6/2010	205	6.476%	4.218%	4.543%	4.179%	-0.32%	**0.04%**

California State Public Works	4/21/2010	246	7.324%	4.771%	5.732%	5.279%	-0.96%	-0.51%
San Francisco Unified School District	5/27/2010	72	5.781%	3.767%	4.273%	3.817%	-0.51%	-0.05%
Los Angeles Department of Water and Power	6/2/2010	616	5.975%	3.890%	4.868%	4.421%	-0.98%	-0.53%
City & County of San Francisco	6/8/2010	193	5.637%	3.678%	4.264%	4.264%	-0.59%	-0.59%
San Francisco City & County Public Utilities	6/17/2010	418	5.863%	3.828%	4.851%	4.441%	-1.02%	-0.61%
Northern California Power Agency	6/24/2010	177	7.357%	4.788%	5.364%	4.993%	-0.58%	-0.21%
Northern California Power Agency	6/24/2010	110	5.691%	3.710%	5.149%	5.149%	-1.44%	-1.44%
San Francisco City & County Public Utilities	8/4/2010	344	5.907%	3.852%	4.097%	4.097%	-0.25%	-0.25%
University of California	9/30/2010	486	5.987%	3.905%	4.684%	4.261%	-0.78%	-0.36%
San Diego County Regional Airport Authority	10/5/2010	215	6.670%	4.341%	4.792%	4.416%	-0.45%	-0.07%
East Bay Municipal Utility District	10/20/2010	150	5.164%	3.364%	4.364%	3.848%	-1.00%	-0.48%
City of Los Angeles	10/21/2010	177	5.749%	3.743%	4.620%	4.150%	-0.88%	-0.41%
Bay Area Toll Authority	11/4/2010	475	6.938%	4.515%	5.027%	4.714%	-0.51%	-0.20%
San Diego County Regional Trans. Commission	11/10/2010	339	6.796%	4.711%	4.928%	4.444%	-0.22%	**0.27%**
Los Angeles County Metropolitan Trans. Authority	11/16/2010	574	5.628%	3.672%	4.586%	4.586%	-0.91%	-0.91%
Santa Clara Valley Transportation Authority	11/17/2010	470	5.802%	3.785%	4.500%	4.500%	-0.72%	-0.72%
University of California	11/18/2010	700	6.490%	4.230%	4.928%	4.928%	-0.70%	-0.70%
Riverside County Transportation Commission	11/30/2010	112	6.849%	4.458%	5.286%	4.803%	-0.83%	-0.35%
City of Newport Beach	11/30/2010	107	6.961%	4.547%	4.948%	4.948%	-0.40%	-0.40%
City of Riverside	12/16/2010	133	7.560%	4.925%	5.847%	5.847%	-0.92%	-0.92%
San Francisco City & County Public Utilities	12/22/2010	351	6.991%	4.548%	5.168%	4.742%	-0.62%	-0.19%
Orange County Local Transportation Authority	12/23/2010	294	6.857%	4.469%	5.153%	5.153%	-0.68%	-0.68%
Totals and Averages		**12,879**	**6.530%**	**4.261%**	**4.904%**	**4.608%**	**-0.64%**	**-0.35%**

Graph 1, California Build America Bonds
California BABs Sold on Same Date as Tax-Exempt Bonds by Same Issuer
Comparison of Actual BABs versus Counterfactual Tax-Exempt



Graph 2, $486,130,000 Regents of the University of California, Limited Project Revenue Bonds, 2010 Series F (BABs)
Comparison of Actual BABs versus Counterfactual Tax-Exempt

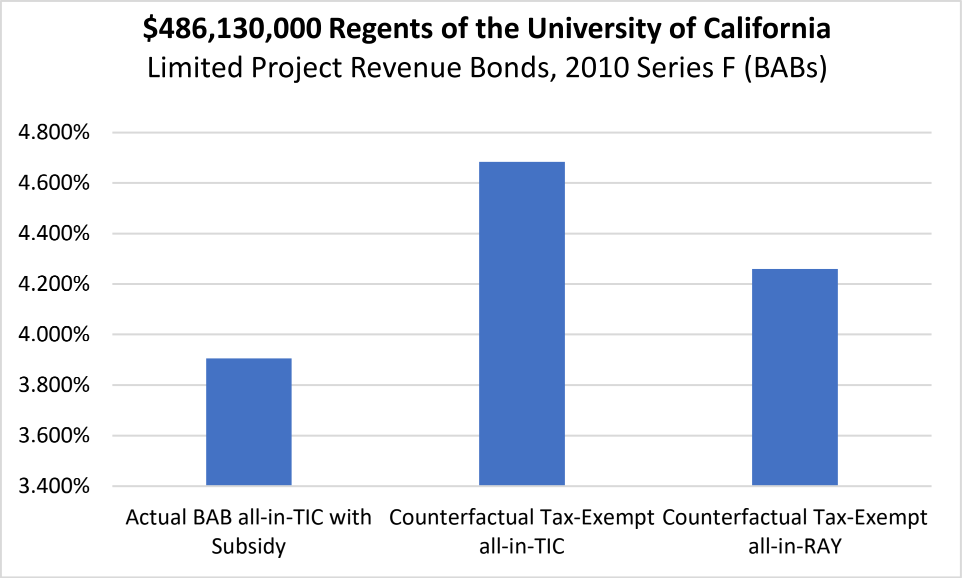


Table 5, California Build America Bonds

California BABs Sold on Same Date as Tax-Exempt Bonds by Same Issuer

Calculation of Neutral Subsidy Rates Based on all-in TIC and all-in RAY

Issuer	Dated Date	(A) Actual BAB all-in TIC without Subsidy	(B) Counter-factual Tax-Exempt all-in TIC	(C) Counter-factual Tax-Exempt all-in RAY	(D) Counter-factual Tax-Exempt all-in TIC as a Percentage of Actual BAB all-in TIC without Subsidy (B)/(A)	(E) Counter-factual Tax-Exempt all-in RAY as a Percentage of Actual BAB all-in TIC without Subsidy (C)/(A)	Neutral Subsidy Rate Between Actual BAB and Counter-Factual Tax-Exempt Based on all-in TIC (100%-D)	Neutral Subsidy Rate Between Actual BAB and Counter-Factual Tax-Exempt Based on all-in RAY (100%-E)
Southern California Metropolitan Water District	6/25/2009	6.196%	4.765%	4.274%	76.90%	68.97%	23.10%	31.03%
Tuolumne Wind Project Authority	7/14/2009	6.963%	5.935%	5.505%	85.25%	79.07%	14.75%	20.93%
Metropolitan Water District of Southern California	8/11/2009	6.387%	4.832%	4.385%	75.66%	68.65%	24.34%	31.35%
Oakland Union School District	8/12/2009	9.555%	6.468%	6.468%	67.69%	67.69%	32.31%	32.31%
Pasadena Unified School District	9/17/2009	7.069%	4.554%	4.081%	64.42%	57.72%	35.58%	42.28%
City & County of San Francisco	10/7/2009	6.498%	4.824%	4.383%	74.24%	67.45%	25.76%	32.55%
Los Angeles Unified School District	10/15/2009	5.793%	4.740%	4.249%	81.83%	73.36%	18.17%	26.64%
California State Public Works	10/29/2009	8.412%	6.018%	6.018%	71.55%	71.55%	28.45%	28.45%
Southwestern Community College	11/5/2009	7.195%	4.925%	4.465%	68.46%	62.05%	31.54%	37.95%
Los Angeles Department of Water and Power	12/3/2009	5.876%	4.708%	4.275%	80.12%	72.74%	19.88%	27.26%
Los Angeles Department of Airports	12/3/2009	6.504%	5.234%	4.763%	80.48%	73.24%	19.52%	26.76%
University of California	12/17/2009	6.610%	4.960%	4.484%	75.03%	67.83%	24.97%	32.17%
City of Riverside	12/22/2009	6.297%	4.636%	4.186%	73.62%	66.47%	26.38%	33.53%
City of Fresno	2/3/2010	6.728%	5.221%	4.865%	77.60%	72.31%	22.40%	27.69%
San Diego County Water Authority	2/4/2010	6.176%	4.845%	4.415%	78.46%	71.49%	21.54%	28.51%
Oxnard Financing Authority	2/11/2010	6.966%	5.145%	4.795%	73.86%	68.84%	26.14%	31.16%
San Mateo Union High School District	2/17/2010	6.671%	4.278%	4.278%	64.13%	64.13%	35.87%	35.87%
Calleguas-Las Virgenes Public Financing Authority	2/24/2010	5.917%	4.038%	4.038%	68.24%	68.24%	31.76%	31.76%
Los Angeles Unified School District	3/4/2010	6.920%	5.041%	4.509%	72.84%	65.15%	27.16%	34.85%

City & County of San Francisco	3/24/2010	5.608%	3.745%	3.745%	66.79%	66.79%	33.21%	33.21%
California State University	4/6/2010	6.476%	4.543%	4.179%	70.15%	64.52%	29.85%	35.48%
California State Public Works	4/21/2010	7.324%	5.732%	5.279%	78.26%	72.08%	21.74%	27.92%
San Francisco Unified School District	5/27/2010	5.781%	4.273%	3.817%	73.91%	66.03%	26.09%	33.97%
Los Angeles Department of Water and Power	6/2/2010	5.975%	4.868%	4.421%	81.48%	74.00%	18.52%	26.00%
City & County of San Francisco	6/8/2010	5.637%	4.264%	4.264%	75.64%	75.64%	24.36%	24.36%
San Francisco City & County Public Utilities	6/17/2010	5.863%	4.851%	4.441%	82.74%	75.75%	17.26%	24.25%
Northern California Power Agency	6/24/2010	7.357%	5.364%	4.993%	72.91%	67.87%	27.09%	32.13%
Northern California Power Agency	6/24/2010	5.691%	5.149%	5.149%	90.47%	90.47%	9.53%	9.53%
San Francisco City & County Public Utilities	8/4/2010	5.907%	4.097%	4.097%	69.36%	69.36%	30.64%	30.64%
University of California	9/30/2010	5.987%	4.684%	4.261%	78.24%	71.17%	21.76%	28.83%
San Diego County Regional Airport Authority	10/5/2010	6.670%	4.792%	4.416%	71.84%	66.20%	28.16%	33.80%
East Bay Municipal Utility District	10/20/2010	5.164%	4.364%	3.848%	84.52%	74.53%	15.48%	25.47%
City of Los Angeles	10/21/2010	5.749%	4.620%	4.150%	80.37%	72.19%	19.63%	27.81%
Bay Area Toll Authority	11/4/2010	6.938%	5.027%	4.714%	72.45%	67.95%	27.55%	32.05%
San Diego County Regional Trans. Commission	11/10/2010	6.796%	4.928%	4.444%	72.52%	65.39%	27.48%	34.61%
Los Angeles County Metropolitan Trans. Authority	11/16/2010	5.628%	4.586%	4.586%	81.50%	81.50%	18.50%	18.50%
Santa Clara Valley Transportation Authority	11/17/2010	5.802%	4.500%	4.500%	77.57%	77.57%	22.43%	22.43%
University of California	11/18/2010	6.490%	4.928%	4.928%	75.94%	75.94%	24.06%	24.06%
Riverside County Transportation Commission	11/30/2010	6.849%	5.286%	4.803%	77.18%	70.13%	22.82%	29.87%
City of Newport Beach	11/30/2010	6.961%	4.948%	4.948%	71.08%	71.08%	28.92%	28.92%
City of Riverside	12/16/2010	7.560%	5.847%	5.847%	77.33%	77.33%	22.67%	22.67%
San Francisco City & County Public Utilities	12/22/2010	6.991%	5.168%	4.742%	73.93%	67.83%	26.07%	32.17%
Orange County Local Transportation Authority	12/23/2010	6.857%	5.153%	5.153%	75.15%	75.15%	24.85%	24.85%
Averages		**6.530%**	**4.904%**	**4.608%**	**75.39%**	**70.78%**	**24.61%**	**29.22%**

Table 6, $250,000,000 Metropolitan Water District of Southern California, Water Revenue Bonds, Series 2009D (Build America Bonds)

Cost of Capital Statistics Taking into Federal Budget Sequester

Issuer	Dated Date	Expected BAB all-in TIC without Subsidy	(A) Expected BAB all-in TIC with Full Subsidy	(B) Actual BAB all-in TIC with Subsidy Adjusted for Budget Sequester	(C) Counter-factual Tax-Exempt all-in TIC	(D) Counter-factual Tax-Exempt all-in RAY	Expected BAB all-in TIC with Full Subsidy minus Counter-factual Tax-Exempt all-in RAY (A) – (D)	Actual BAB all-in TIC with Subsidy Adjusted for Budget Sequester minus Counter-factual Tax-Exempt all-in RAY (B) – (D)
Metropolitan Water District of Southern California	8/11/2009	6.387%	4.165%	4.211%	4.832%	4.385%	-0.220%	-0.174%

Table 7, $250,000,000 Metropolitan Water District of Southern California, Water Revenue Bonds, Series 2009D (Build America Bonds)

Debt Service Statistics Taking into Federal Budget Sequester (debt service in $)

Expected BAB all-in TIC without Subsidy	Expected BAB Debt Service without Subsidy	Expected BAB all-in TIC with Full Subsidy	Expected BAB Debt Service with Full Subsidy	Actual BAB all-in TIC with Subsidy Adjusted for Budget Sequester	Actual BAB Debt Service with Subsidy Adjusted for Budget Sequester	Counter-factual Tax-Exempt all-in TIC	Counter-factual Tax-Exempt all-in TIC Debt Service	Counter-factual Tax-Exempt all-in RAY	Counter-factual Tax-Exempt all-in RAY Debt Service
6.387%	576,576,178	4.165%	462,274,515	4.211%	464,251,941	4.832%	506,017,118	4.385%	469,823,584
						Difference from Expected BAB Debt Service with Full Subsidy	43,742,602		7,549,068
						Difference from Expected BAB Debt Service with Subsidy Adjusted for Budget Sequester	41,765,177		5,571,643

Table 8, California Build America Bonds
California BABs Sold on Same Date as Tax-Exempt Bonds by Same Issuer

Calculation of Neutral Subsidy Rates Based on all-in RAY, Current Refundings (CR) versus Advance Refundings (AR)

Issuer	(A) Actual BAB all-in TIC without Subsidy	(B) Counter-factual Tax-Exempt all-in RAY (CR)	(C) Counter-factual Tax-Exempt all-in RAY (AR)	Difference between Counter-factual all-in Ray (CR) and Counter-factual all-in RAY (AR) (B-C)	(D) Counter-factual Tax-Exempt all-in RAY (CR) as a Percentage of Actual BAB all-in TIC without Subsidy (B)/(A)	(E) Counter-factual Tax-Exempt all-in RAY (AR) as a Percentage of Actual BAB all-in TIC without Subsidy (C)/(A)	Neutral Subsidy Rate Between Actual BAB and Counter-Factual Tax-Exempt Based on all-in RAY (CR) (100%-D)	Neutral Subsidy Rate Between Actual BAB and Counter-Factual Tax-Exempt Based on all-in RAY (AR) (100%-E)
Southern California Metropolitan Water District	6.196%	4.274%	4.239%	-0.035%	68.97%	68.41%	31.03%	31.59%
Tuolumne Wind Project Authority	6.963%	5.505%	5.485%	-0.021%	79.07%	78.77%	20.93%	21.23%
Metropolitan Water District of Southern California	6.387%	4.385%	4.374%	-0.010%	68.65%	68.49%	31.35%	31.51%
Oakland Union School District	9.555%	6.468%	6.468%	0.000%	67.69%	67.69%	32.31%	32.31%
Pasadena Unified School District	7.069%	4.081%	4.062%	-0.019%	57.72%	57.46%	42.28%	42.54%
City & County of San Francisco	6.498%	4.383%	4.349%	-0.034%	67.45%	66.93%	32.55%	33.07%
Los Angeles Unified School District	5.793%	4.249%	4.224%	-0.026%	73.36%	72.91%	26.64%	27.09%
California State Public Works	8.412%	6.018%	6.018%	0.000%	71.55%	71.55%	28.45%	28.45%
Southwestern Community College	7.195%	4.465%	4.432%	-0.032%	62.05%	61.61%	37.95%	38.39%
Los Angeles Department of Water and Power	5.876%	4.275%	4.256%	-0.019%	72.74%	72.42%	27.26%	27.58%
Los Angeles Department of Airports	6.504%	4.763%	4.744%	-0.019%	73.24%	72.94%	26.76%	27.06%
University of California	6.610%	4.484%	4.450%	-0.033%	67.83%	67.33%	32.17%	32.67%
City of Riverside	6.297%	4.186%	4.161%	-0.025%	66.47%	66.08%	33.53%	33.92%
City of Fresno	6.728%	4.865%	4.841%	-0.024%	72.31%	71.95%	27.69%	28.05%
San Diego County Water Authority	6.176%	4.415%	4.373%	-0.042%	71.49%	70.80%	28.51%	29.20%
Oxnard Financing Authority	6.966%	4.795%	4.769%	-0.026%	68.84%	68.47%	31.16%	31.53%
San Mateo Union High School District	6.671%	4.278%	4.278%	0.000%	64.13%	64.13%	35.87%	35.87%
Calleguas-Las Virgenes Public Financing Authority	5.917%	4.038%	4.038%	0.000%	68.24%	68.24%	31.76%	31.76%

Los Angeles Unified School District	6.920%	4.509%	4.478%	-0.031%	65.15%	64.71%	34.85%	35.29%
City & County of San Francisco	5.608%	3.745%	3.745%	0.000%	66.79%	66.79%	33.21%	33.21%
California State University	6.476%	4.179%	4.161%	-0.018%	64.52%	64.25%	35.48%	35.75%
California State Public Works	7.324%	5.279%	5.275%	-0.004%	72.08%	72.02%	27.92%	27.98%
San Francisco Unified School District	5.781%	3.817%	3.819%	0.002%	66.03%	66.07%	33.97%	33.93%
Los Angeles Department of Water and Power	5.975%	4.421%	4.377%	-0.044%	74.00%	73.27%	26.00%	26.73%
City & County of San Francisco	5.637%	4.264%	4.264%	0.000%	75.64%	75.64%	24.36%	24.36%
San Francisco City & County Public Utilities	5.863%	4.441%	4.417%	-0.023%	75.75%	75.35%	24.25%	24.65%
Northern California Power Agency	7.357%	4.993%	4.969%	-0.024%	67.87%	67.55%	32.13%	32.45%
Northern California Power Agency	5.691%	5.149%	5.149%	0.000%	90.47%	90.47%	9.53%	9.53%
San Francisco City & County Public Utilities	5.907%	4.097%	4.097%	0.000%	69.36%	69.36%	30.64%	30.64%
University of California	5.987%	4.261%	4.226%	-0.035%	71.17%	70.58%	28.83%	29.42%
San Diego County Regional Airport Authority	6.670%	4.416%	4.393%	-0.022%	66.20%	65.87%	33.80%	34.13%
East Bay Municipal Utility District	5.164%	3.848%	3.810%	-0.038%	74.53%	73.79%	25.47%	26.21%
City of Los Angeles	5.749%	4.150%	4.114%	-0.036%	72.19%	71.57%	27.81%	28.43%
Bay Area Toll Authority	6.938%	4.714%	4.683%	-0.032%	67.95%	67.49%	32.05%	32.51%
San Diego County Regional Trans. Commission	6.796%	4.444%	4.404%	-0.040%	65.39%	64.80%	34.61%	35.20%
Los Angeles County Metropolitan Trans. Authority	5.628%	4.586%	4.586%	0.000%	81.50%	81.50%	18.50%	18.50%
Santa Clara Valley Transportation Authority	5.802%	4.500%	4.500%	0.000%	77.57%	77.57%	22.43%	22.43%
University of California	6.490%	4.928%	4.928%	0.000%	75.94%	75.94%	24.06%	24.06%
Riverside County Transportation Commission	6.849%	4.803%	4.764%	-0.038%	70.13%	69.57%	29.87%	30.43%
City of Newport Beach	6.961%	4.948%	4.948%	0.000%	71.08%	71.08%	28.92%	28.92%
City of Riverside	7.560%	5.847%	5.847%	0.000%	77.33%	77.33%	22.67%	22.67%
San Francisco City & County Public Utilities	6.991%	4.742%	4.697%	-0.045%	67.83%	67.18%	32.17%	32.82%
Orange County Local Transportation Authority	6.857%	5.153%	5.153%	0.000%	75.15%	75.15%	24.85%	24.85%
Averages	**6.530%**	**4.608%**	**4.590%**	**-0.018%**	**70.78%**	**70.49%**	**29.22%**	**29.51%**

10. Appendix

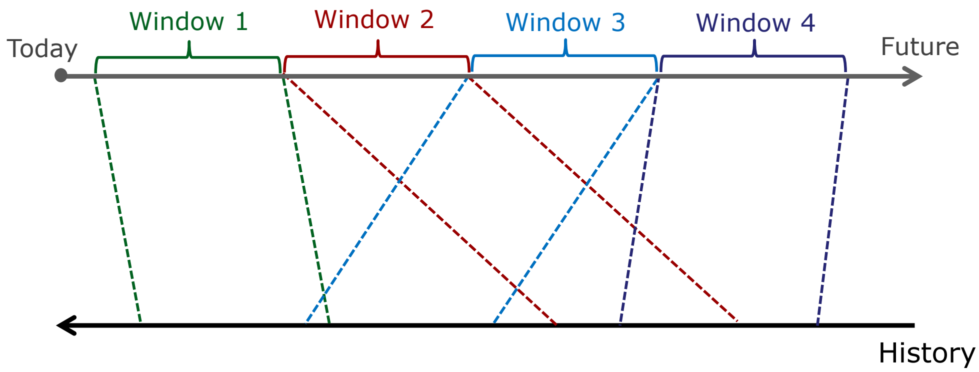
This appendix reviews the market model used to generate real world and history-consistent yield curves from markets relevant to the issuer's refunding decision: issuer tax-exempt, issuer taxable, and U.S. Treasury issued State and Local Government Securities (SLGS). Deguillaume, Rebonato and Pogudin (2013) provides a complete treatment. Though not intended to be an exhaustive description, we hope it provides both some intuition behind the modeling approach and, at minimum, evidence of success, namely that the simulation reliably creates long-term, history-consistent municipal, taxable, and escrow market simulations. The reason for the success of Deguillaume et al (2013) model is that it identifies a universal relationship in how interest rates have changed historically:

> "We show that the most convincing explanation for the observed dependence of the magnitude of rate changes on the level of rates is a function with three 'regimes': when rates are very low (say, below 1.5%), changes appear to be proportional to the level. For rates between, approximately, 1.5% and 5% changes appear to be independent of the level of rates; above 5% we find that proportionality prevails again, although possibly with a different 'slope'. The relationship that we empirically find is surprisingly robust across currencies, rate maturities and time periods. Indeed, we show that we find the same three-regime relationship (although with somewhat different break points) even using UK Consol yields going back to the XIX century!"

Deguillaume et al (2013) method is a combination of parametric modeling and simulation by historical sampling. Historic yield changes are normalized according to a well-defined mapping that recognizes the existence of the three regimes, then sampled, and then rescaled to match the current regime. This has the effect of producing credible interest rate simulations while at the same time allowing for parametric regime switching between three persistent regimes.

The Market Simulation Approach

The approach to creating simulated market environments involves historical sampling. We use a window of historic data and the yield changes therein to simulate rate changes going into the future. For example, a window length of 100 days might be used to sample from ten years of historic daily data. The start point for each window is randomly selected. In order to properly capture the intra and inter-market



correlations between different tenors, we model all yield changes for each tenor using the same selection of windows.

One of the benefits of this approach is it captures the tendency for yields to move in trends (i.e., serial autocorrelation). A challenge occurs at the short end of the yield curve where, due to central bank management of business cycles, autocorrelation may last for years. However, using a long window on the order of years may affect the randomness of the scheme, depending in part on the size of the data set used. Our starting point for the model uses a window length of 100 days.[16] Later we show that our results are robust to different window specifications.

Some Market Model Details

In order to implement the regime-switching, Deguillaume et al (2013) define a class of volatility function for yields (y) which incorporates three regimes based upon rate level,

$$\begin{aligned} \sigma(y) &= \sigma_G \frac{y}{y_L} && 0 < y < y_L \\ &= \sigma_G && y_L \leq y < y_R \\ &= \sigma_G[1 + K(y - y_R)] && y_R < y \end{aligned}$$

The parameters y_L, y_R, and K can be calibrated using historical interest rate data. The terms y_L and y_R are the lower and upper rate thresholds respectively indicating where the breaks occur between normal and (approximately) lognormal regions. K is the slope of the line when rates are in the high region. Note that σ cannot become negative and is bounded at zero.

[16] See (R. Rebonato, S. Mahal, et al. 2005) for more information on window selection in this setting.

In order to perform this calibration for the municipal market we replicated the work of Deguillaume et al (2013) and applied it to daily tax-exempt data going back to 1964 shown in Figures 1a and 1b. Our results in the US municipal market confirm that the calibration method is more stable for values of y_L but less so for values of y_R and K. That said and as we will show, we find a good fit for all tenors in both tax-exempt and U.S. Treasury markets by using the following values:[17]

Parameter	Value
y_L	1.50%
y_R	6.00%
K	25.0

To generate simulations we randomly sample start dates for windows of historical yield changes, transform the data using what Deguillaume, et al call a ∑ function (a bijection and in some sense a discretized version of the σ function above), and add those transformed changes to the current market to create a simulated real-world environment into the future.

Consider the ∑ function a type of generalized logarithm, which handles more complex rate variations than a simple percentage change of rate level. In essence, the ∑ function achieves a type of disguise. That is, after the ∑ transformation is applied it is very difficult to tell in which type of rate environment the change actually occurred. In short, it is a type of normalization of interest rate variation that allows all data to be treated equally, in what the authors call the "additive world." This is an extremely powerful result and remarkably convenient for generating real-world interest rate simulations. In the case of issuers and investors, the real-world market needs modeling.[18]

Reversion

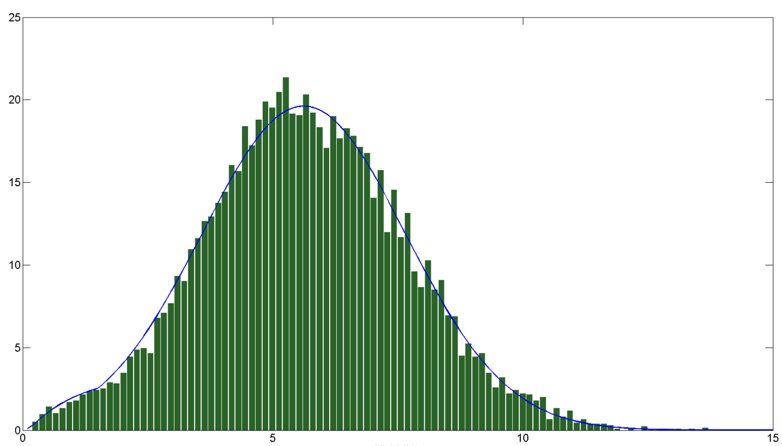
The model described by Deguillaume et al (2013) provides for both a level reversion mechanism for the longest (least volatile) yield as well as a shape median reversion for all other tenors. The former

[17] Recall the issuer taxable curves are a fixed spread in all simulations over the U.S. Treasury curve.
[18] Without getting bogged down in measure theory and the Fundamental Theorem of Asset Pricing, note that in a setting where a financial position can and **is** hedged, so-called "risk-neutral" valuation and management techniques embodied in standard no-arbitrage models beautifully apply. However, the case of unhedged municipal bonds (issuers *and* investors) require real-world measure and risk-preferences.

keeps the long tenor bounded and in the range of reasonable rates like any mean-reverting yield curve model and the latter keeps the shapes of the simulated curves from becoming unrealistic over time. Each of these values is calibrated from historical data. In our analysis, tax-exempt yields include thirteen tenors: 3 month and 1-5, 7, 10, 12, 15, 20, 30, and 40 years. The SLGS component has 3 and 6-month tenors, as well as 1-5, 7, 10, 12, 15, 20, and 30 years, 13 in all. In standard yield curve, modeling parlance this would be considered a 26-factor model. Calibrating to the data set, we find annual median reversion and shape median reversion to be 12% and 72% respectively. Tax-exempt and SLGS yield curves towards which rates will revert in the model are shown in Figures 2a and 2b.

We have also developed an analytic approximation for rate distributions in the model, which is convenient for confirming results of the calibration. This analytic approximation is not defining a separate distribution and fitting it to the model, but rather using the changes of rates as the distribution itself. Figure 3 shows the distribution of the 30-year SLGS rate in 30 years relative to a histogram of 10,000 simulated 30-year SLGS rates.

Figure 3
SLGS 30 Year Tenor at 30 Year horizon



Figures 4a-d plot the .5th, 2nd, 5th, 25th, 50th, 75th, 95th, 98th and 99.5th percentile at various points in time between zero and thirty years comparing an analytic approximation of the distribution of the 2, 5, 10, and 30 year tenors with the same percentiles from the simulated data. These figures were generated using the following inputs:

Number of Simulations	5,000
Simulation Horizon	30 Years
Window Length	100 Days
Long tenor reversion speed (annual)	11.8%
Shape reversion speed (annual)	71.7%
Data Increment	1 Business Day

We see a very close correspondence between analytic approximations of the distribution based upon the sample data itself and simulated distributions throughout the horizon of the analysis. Figures 5a-d provide the same information for the U.S. Treasury/SLGS data. Again, we see a very close match between analytic approximations of the distributions and the simulated data, even at the fat-tail extremes of the .5th and 99.5th percentiles.

The most difficult visual test for a yield curve model like this, particularly one that spans multiple markets, is to see how tenors move together i.e. the plausibility of all market yield curves actually generated within the model. In Figure 6, we show 10 randomly generated market yield curves for two markets at the 30-year time point. The top half shows tax-exempt issuer yield curves comprised of the 13 tenors used in the tax-exempt market while the bottom half shows the 13 tenors used in the SLGS simulation.[19] Notice that the line colors between the two graphs reflect a simulation from the same market environment. For example, the top green line in the tax-exempt graph corresponds to the market environment of the green SLGS yield curve in the bottom half of the Figure. Even with this small number of samples, we can see the greater variability in the shape of the SLGS curves.

[19] One of the benefits of this model is that matching tenors across markets is not a requirement. The historical sampling method is indifferent to the data selected.

Figures

Figure 1a

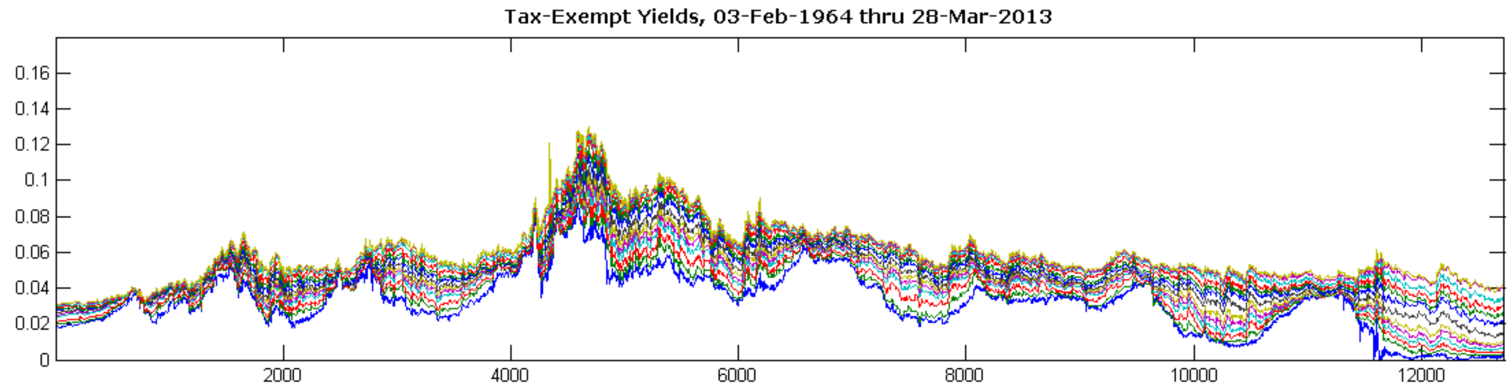


Figure 1b

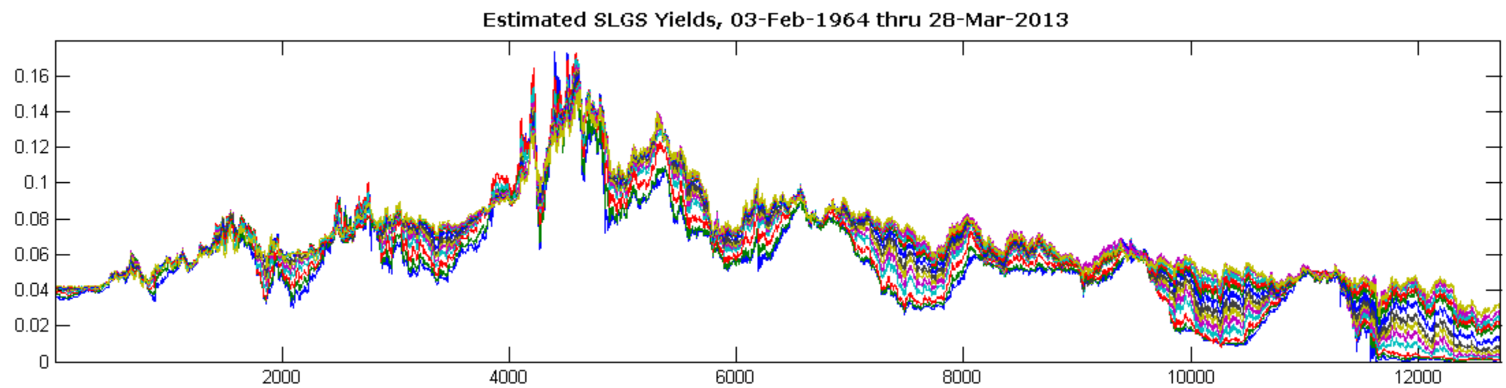


Figure 2a
Tax-exempt Starting and Long Horizon Yields, Base Case

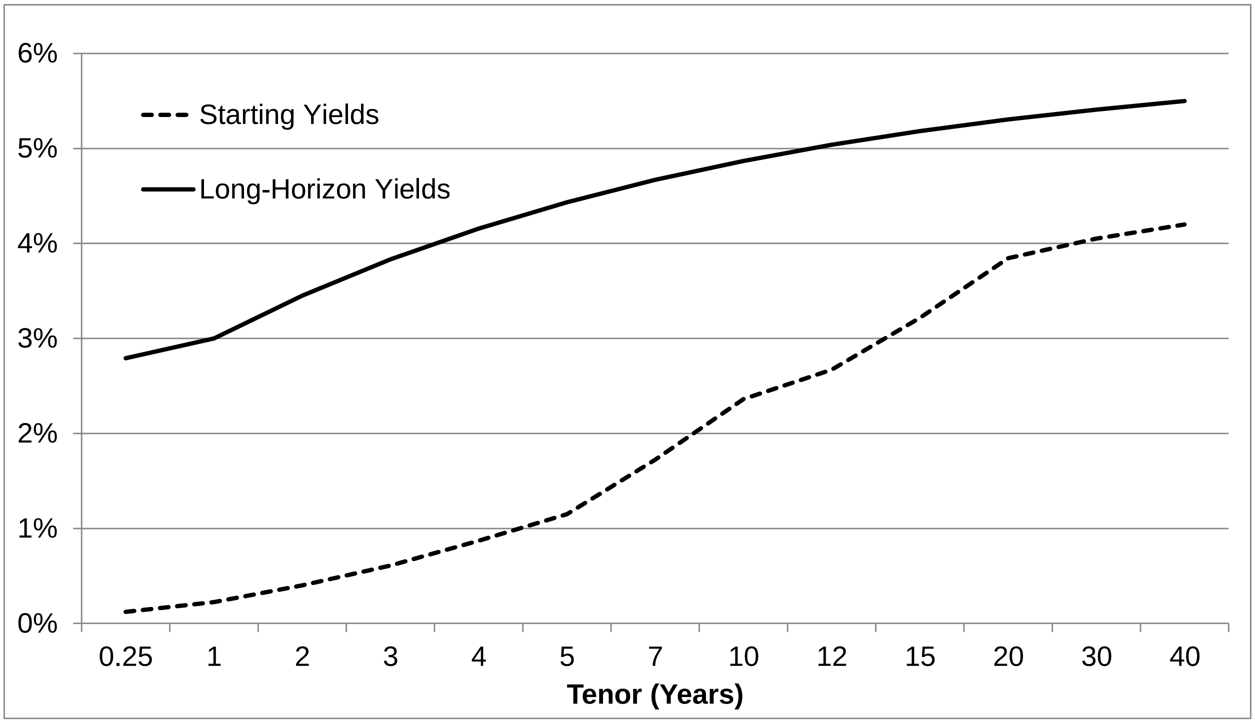


Figure 2b
SLGS Starting and Long Horizon Yields, Base Case

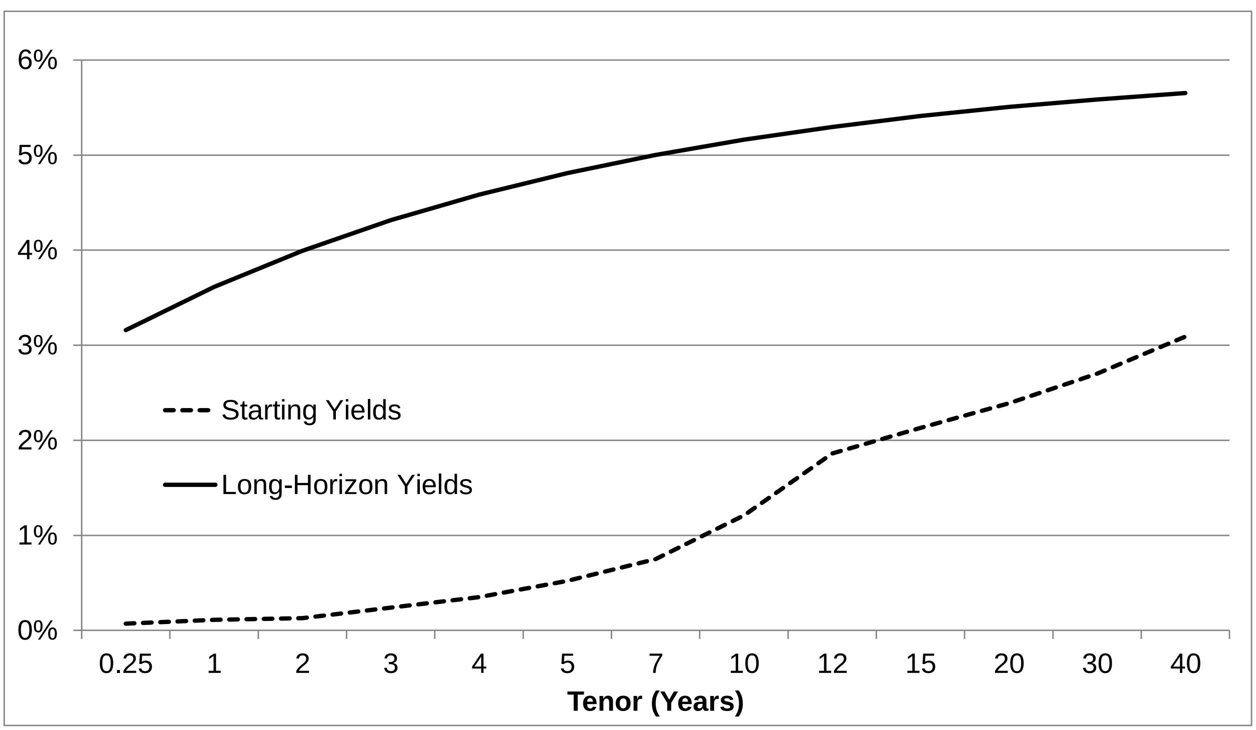


Figure 3

SLGS 30 Year Tenor at 30 Year Horizon

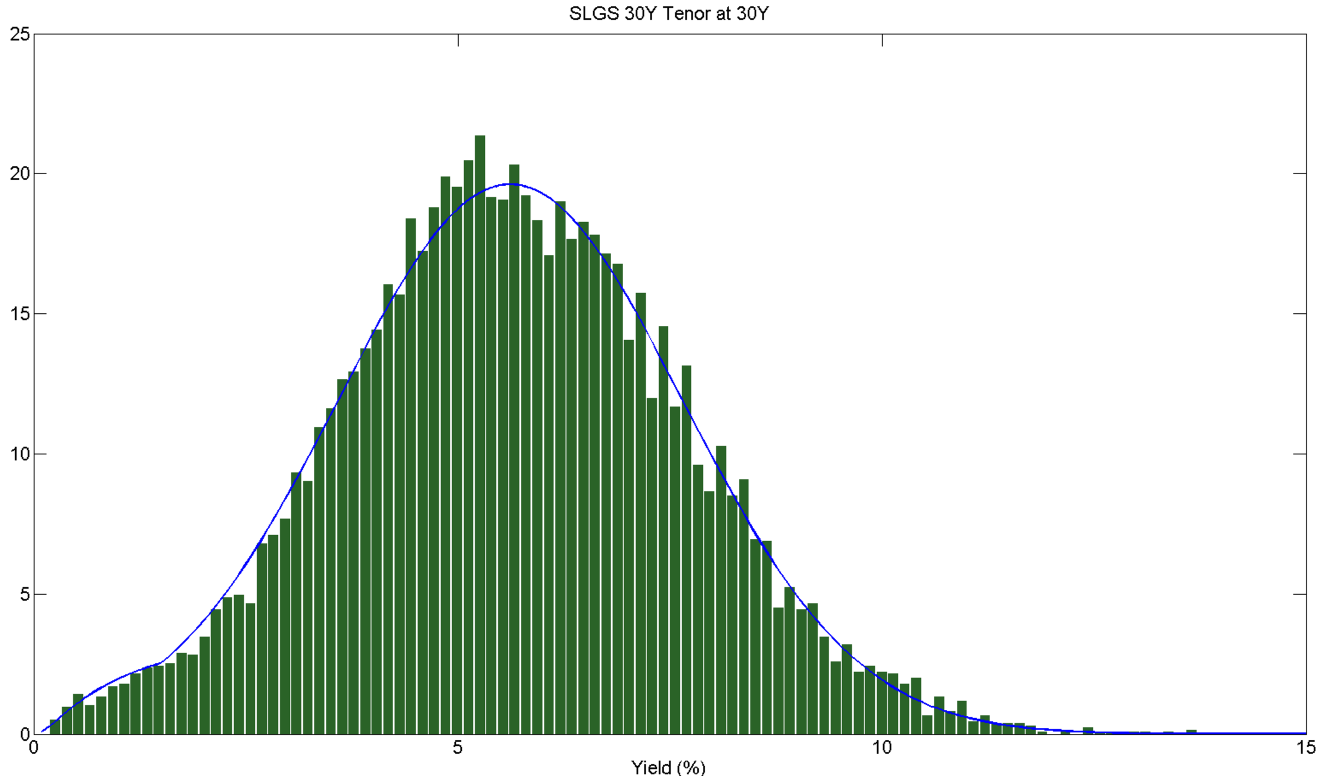


Figure 4a

30Y Tax-exempt Yield – Simulated vs Analytic Approximation over 30 Years

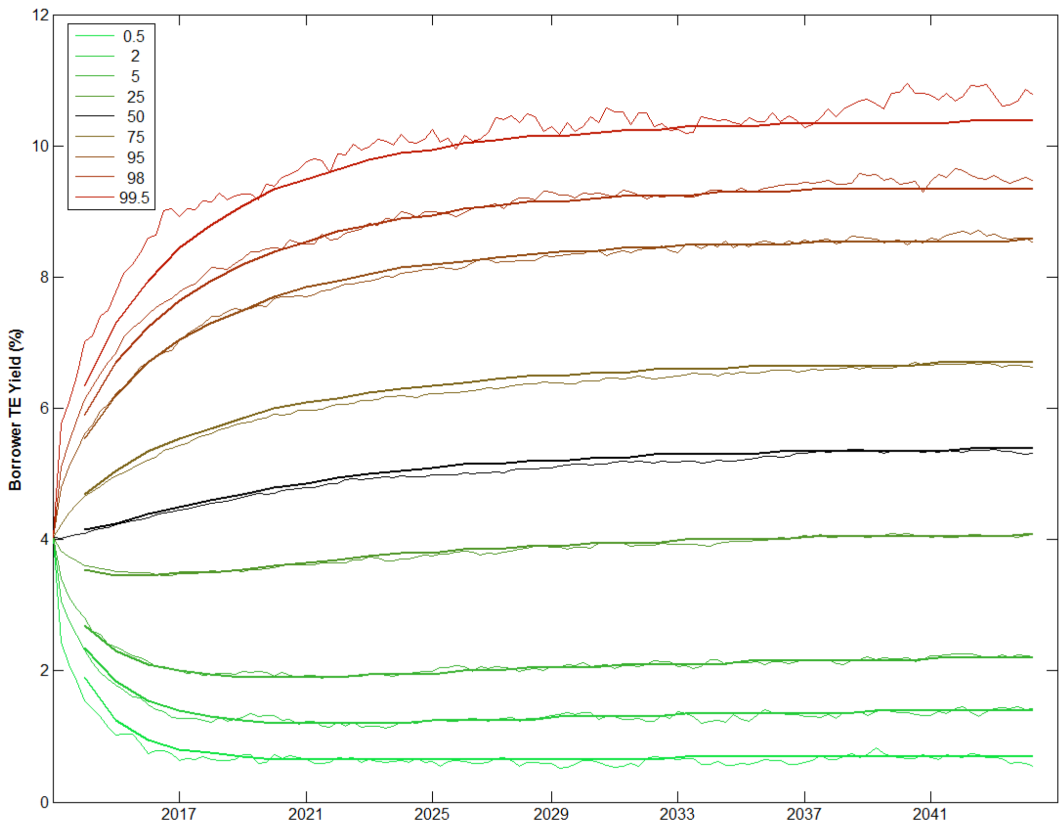


Figure 4b
10Y Tax-exempt Yield – Simulated vs Analytic Approximation over 30 Years

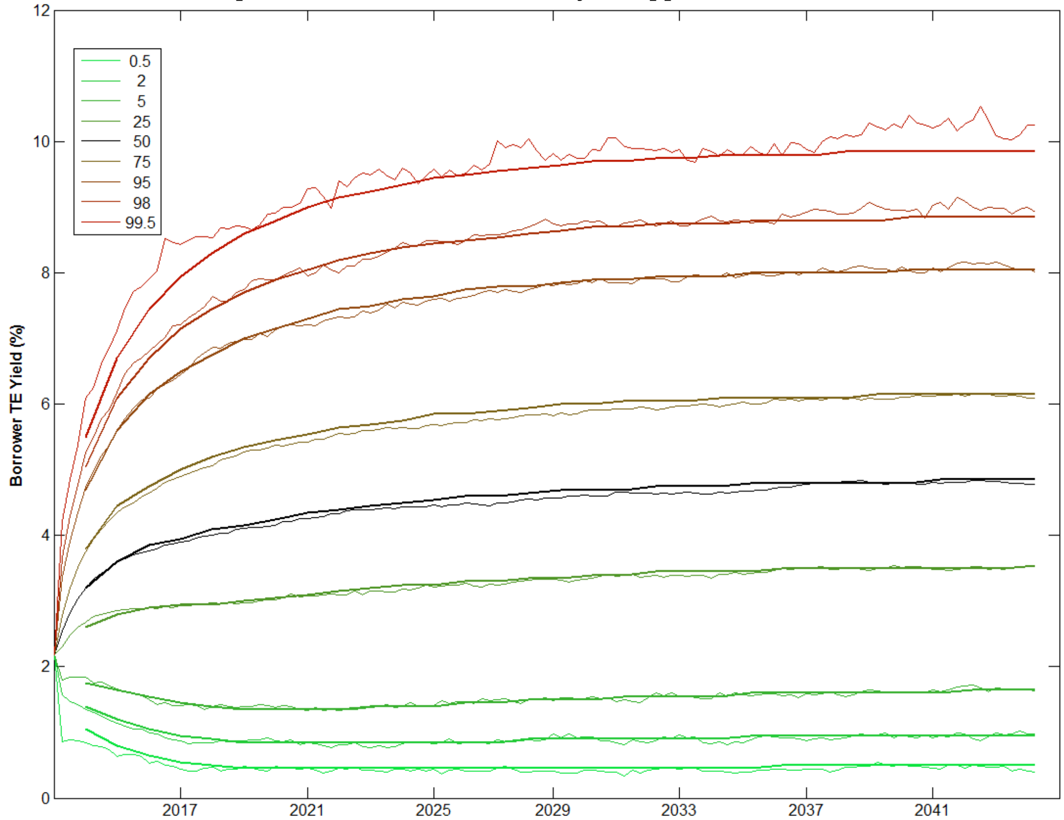


Figure 4c
5Y Tax-exempt Yield – Simulated vs Analytic Approximation over 30 Years

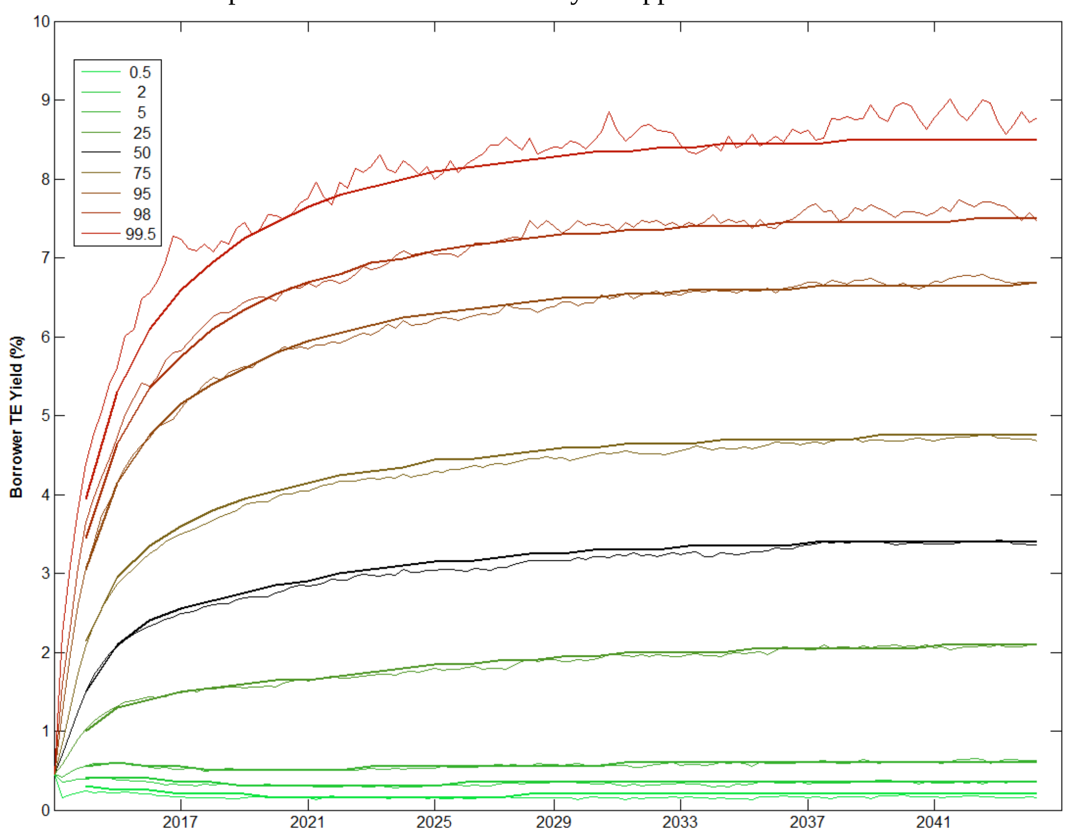


Figure 4d
2Y Tax-exempt Yield – Simulated vs Analytic Approximation over 30 Years

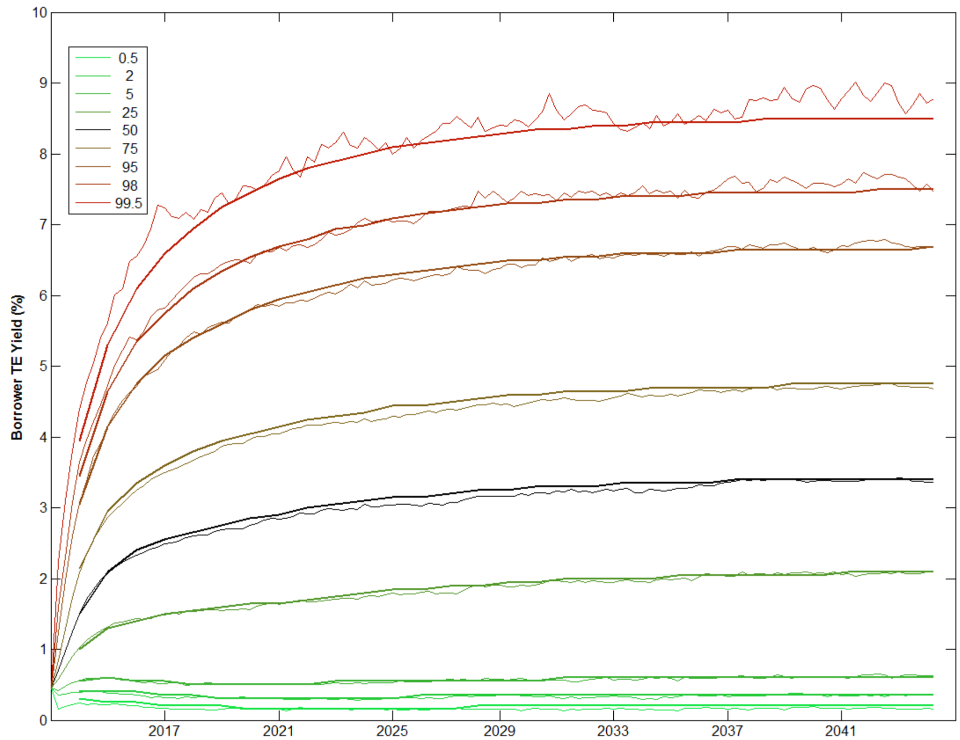


Figure 5a
30Y SLGS Yield – Simulated vs Analytic Approximation over 30 Years

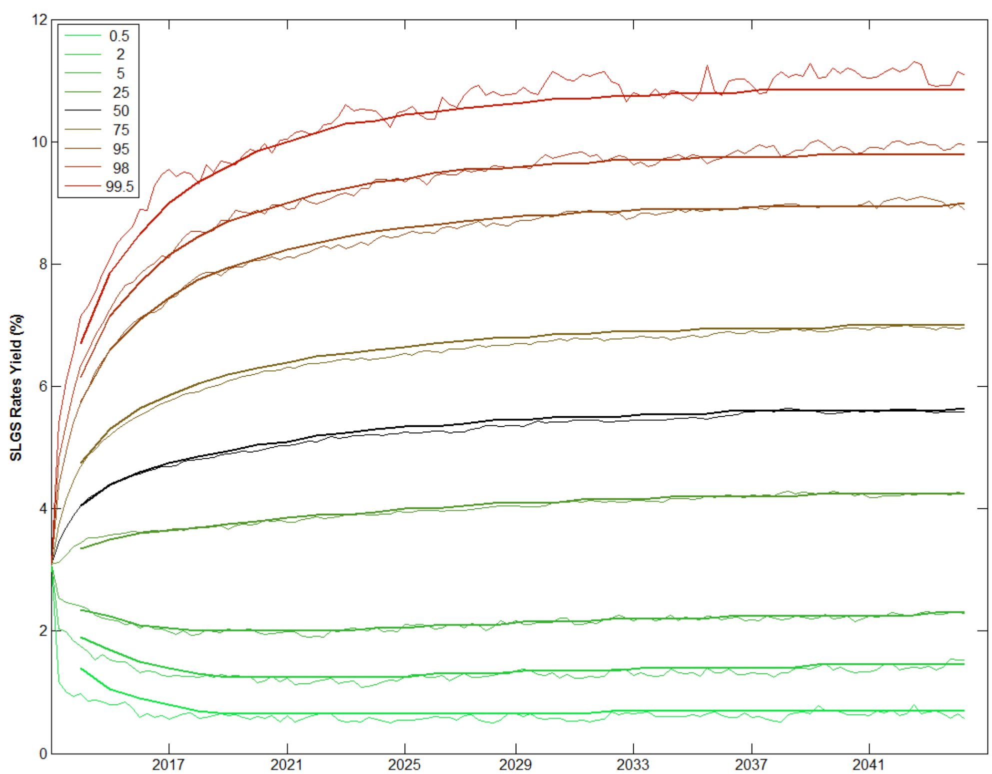


Figure 5b

10Y SLGS Yield – Simulated vs Analytic Approximation over 30 Years

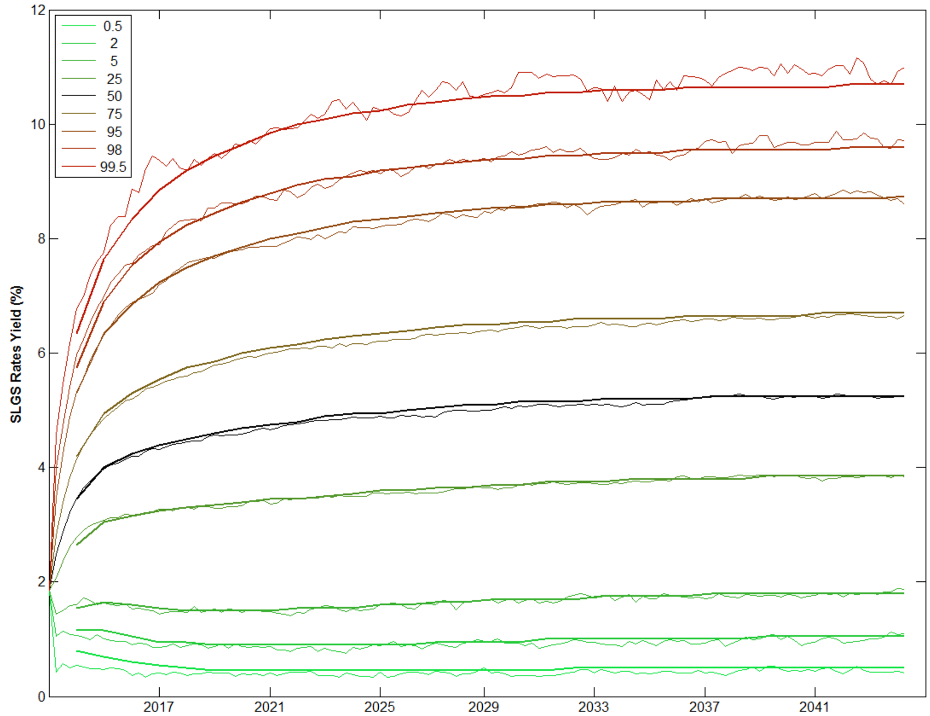


Figure 5c

5Y SLGS Yield – Simulated vs Analytic Approximation over 30 Years

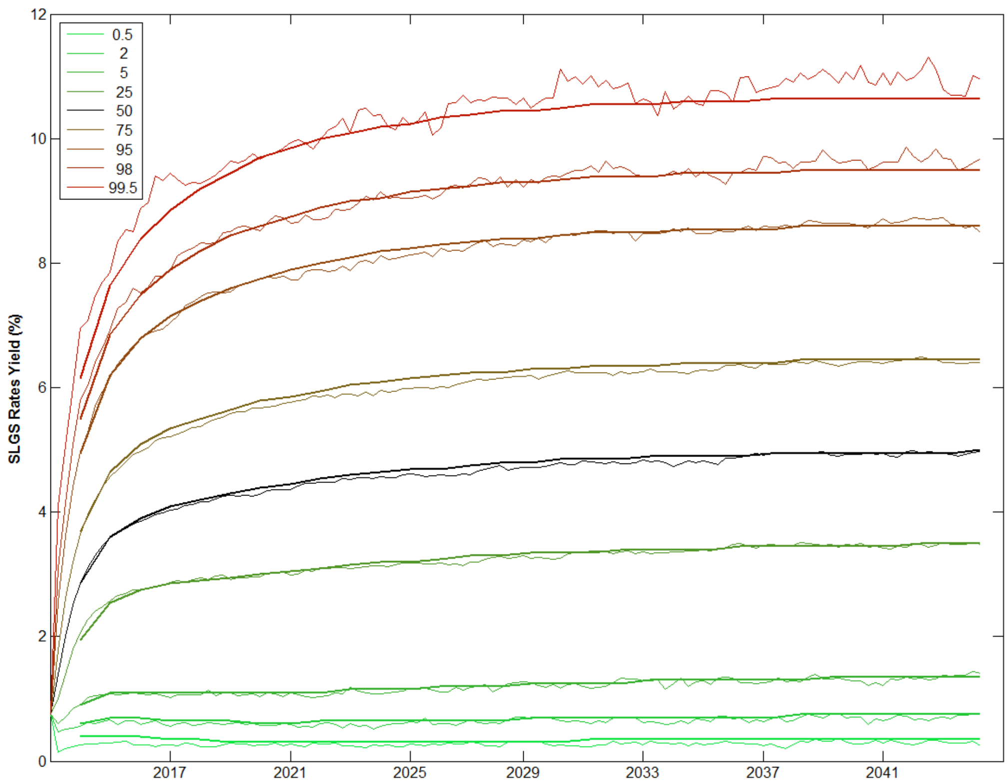


Figure 5d
2Y SLGS Yield – Simulated vs Analytic Approximation over 30 Years

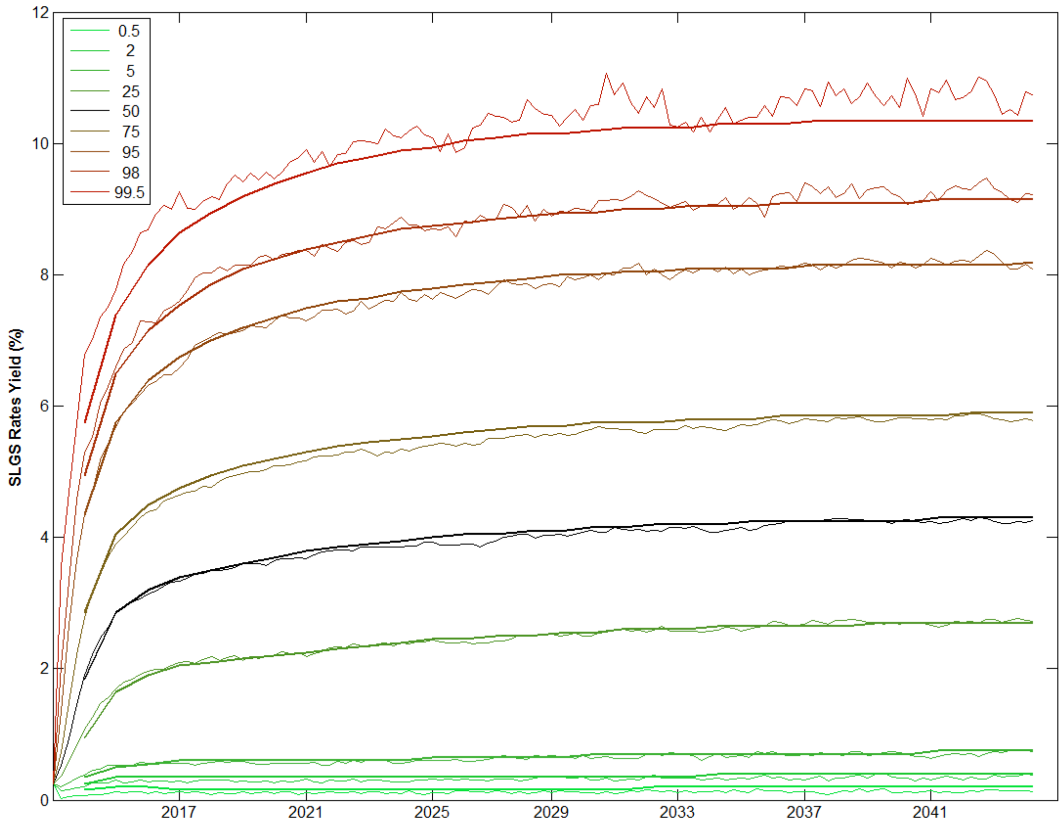


Figure 6
Simulated Tax-exempt and SLGS Yields, 30 Year Horizon